K^0

$I(J^P) = \frac{1}{2}(0^-)$

K^0 MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
497.648±0.022 OUR FIT				
497.648±0.022 OUR AVERAGE				
497.625±0.001±0.031	655k	LAI 02	NA48	K^0_L beam
497.661±0.033	3713	BARKOV 87B	CMD	$e^+e^- \to K^0_L K^0_S$
497.742±0.085	780	BARKOV 85B	CMD	$e^+e^- \to K^0_L K^0_S$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
497.44 ±0.50		FITCH 67	OSPK	
498.9 ±0.5	4500	BALTAY 66	HBC	K^0 from $\overline{p}p$
497.44 ±0.33	2223	KIM 65B	HBC	K^0 from $\overline{p}p$
498.1 ±0.4		CHRISTENS... 64	OSPK	

$m_{K^0} - m_{K^\pm}$

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
3.972±0.027 OUR FIT Error includes scale factor of 1.2.					
• • • We do not use the following data for averages, fits, limits, etc. • • •					
3.95 ±0.21	417	HILL 68B	DBC	+	$K^+d \to K^0pp$
3.90 ±0.25	9	BURNSTEIN 65	HBC	−	
3.71 ±0.35	7	KIM 65B	HBC	−	$K^-p \to n\overline{K}^0$
5.4 ±1.1		CRAWFORD 59	HBC	+	
3.9 ±0.6		ROSENFELD 59	HBC	−	

K^0 MEAN SQUARE CHARGE RADIUS

VALUE (fm^2)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
−0.077±0.010 OUR AVERAGE				
−0.077±0.007±0.011	5037	ABOUZAID 06	KTEV	$K^0_L \to \pi^+\pi^-e^+e^-$
−0.090±0.021		LAI 03C	NA48	$K^0_L \to \pi^+\pi^-e^+e^-$
−0.054±0.026		MOLZON 78		K_S regen. by electrons
• • • We do not use the following data for averages, fits, limits, etc. • • •				
−0.087±0.046		BLATNIK 79		VMD + dispersion relations
−0.050±0.130		FOETH 69B		K_S regen. by electrons

T-VIOLATION PARAMETER IN K^0-$\overline{K}^0$ MIXING

The asymmetry $A_T = \frac{\Gamma(\overline{K}^0 \to K^0) - \Gamma(K^0 \to \overline{K}^0)}{\Gamma(\overline{K}^0 \to K^0) + \Gamma(K^0 \to \overline{K}^0)}$ must vanish if *T* invariance holds.

ASYMMETRY A_T IN K^0-$\overline{K}^0$ MIXING

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*	*TECN*
6.6±1.3±1.0	640k	[1] ANGELOPO... 98E	CPLR

[1] ANGELOPOULOS 98E measures the asymmetry $A_T = [\Gamma(\overline{K}^0_{t=0} \to e^+ \pi^- \nu_{t=\tau}) - \Gamma(K^0_{t=0} \to e^- \pi^+ \overline{\nu}_{t=\tau})]/[\Gamma(\overline{K}^0_{t=0} \to e^+ \pi^- \nu_{t=\tau}) + \Gamma(K^0_{t=0} \to e^- \pi^+ \overline{\nu}_{t=\tau})]$ as a function of the neutral-kaon eigentime τ. The initial strangeness of the neutral kaon is tagged by the charge of the accompanying charged kaon in the reactions $p\overline{p} \to K^- \pi^+ K^0$ and $p\overline{p} \to K^+ \pi^- \overline{K}^0$. The strangeness at the time of the decay is tagged by the lepton charge. The reported result is the average value of A_T over the interval $1\tau_S < \tau < 20\tau_S$. From this value of A_T ANGELOPOULOS 01B, assuming *CPT* invariance in the $e\pi\nu$ decay amplitude, determine the *T*-violating as $\Delta S = \Delta S$ conserving parameter (for its definition, see Review below) $4\text{Re}(\epsilon) = (6.2 \pm 1.4 \pm 1.0) \times 10^{-3}$.

CPT INVARIANCE TESTS IN NEUTRAL KAON DECAY

Revised June 2006 by P. Bloch (CERN).

The time evolution of a neutral kaon state state is described by

$$\frac{d}{dt}\Psi = -i\Lambda\Psi \ , \qquad \Lambda \equiv M - \frac{i}{2}\Gamma \tag{1}$$

where M and Γ are Hermitian 2×2 matrices known as the mass and decay matrices. The corresponding eigenvalues are $\lambda_{L,S} = m_{L,S} - \frac{i}{2}\gamma_{L,S}$. *CPT* invariance requires the diagonal elements of Λ to be equal. The *CPT*-violation complex parameter δ is defined as

$$\begin{aligned}\delta &= \frac{\Lambda_{\overline{K}^0\overline{K}^0} - \Lambda_{K^0K^0}}{2(\lambda_L - \lambda_S)} \\ &= \delta_{\parallel}\exp(i\phi_{SW}) + \delta_{\perp}\exp(i(\phi_{SW} + \frac{\pi}{2}))\end{aligned} \tag{2}$$

where we have introduced the projections $\delta_{\parallel}$ and $\delta_{\perp}$ respectively parallel and perpendicular to the superweak direction $\phi_{SW} = \tan^{-1}(2\Delta m/\Delta\gamma)$, where $\Delta m = m_L - m_S$ and $\Delta\gamma = \gamma_S - \gamma_L$, the positive mass and width differences between K_L and K_S. These projections are linked to the mass and width difference between K^0 and $\overline{K}^0$:

$$\delta_{\parallel} = \frac{1}{4}\frac{\gamma_{K^0} - \gamma_{\overline{K}^0}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\gamma}{2}\right)^2}} \ , \ \delta_{\perp} = \frac{1}{2}\frac{m_{K^0} - m_{\overline{K}^0}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\gamma}{2}\right)^2}} \ . \tag{3}$$

$\mathrm{Re}(\delta)$ can be directly measured by studying the time evolution of the strangeness content of initially pure K^0 and $\overline{K}^0$ states, for example through the asymmetry

$$A_{CPT} = \frac{P[\overline{K}^0 \to \overline{K}^0(t)] - P[K^0 \to K^0(t)]}{P[\overline{K}^0 \to \overline{K}^0(t)] + P[K^0 \to K^0(t)]} = 4\mathrm{Re}(\delta) \qquad (4)$$

where $P[a \to b(t)]$ is the probability that the pure initial state a is seen as state b at proper time t. This method has been used by tagging the initial strangeness with strong interactions and the final strangeness with the semileptonic decay (a more appropriate combination of semileptonic rates allows to be independent of any direct CPT violation in the decay itself) and yields today's best value of $\mathrm{Re}(\delta)$, compatible with zero with an error of $\sim 3 \times 10^{-4}$.

As an alternative it has been proposed to compare the semileptonic charge asymmetries for K_L and K_S

$$\begin{aligned} A_{L,S} =& \frac{R(K_{L,S} \to \pi^-\ell^+\nu) - R(K_{L,S} \to \pi^+\ell^-\overline{\nu})}{R(K_{L,S} \to \pi^-\ell^+\nu) + R(K_{L,S} \to \pi^+\ell^-\overline{\nu})} \ , \\ & A_S - A_L = 4\mathrm{Re}(\delta) \ . \qquad (5) \end{aligned}$$

A_L has been accurately measured. A_S has been recently measured with tagged K_S at ϕ factories, however not yet with the required accuracy. Note however that Eq. (5) assumes CPT invariance in the $\Delta S = -\Delta Q$ semileptonic decay amplitude.

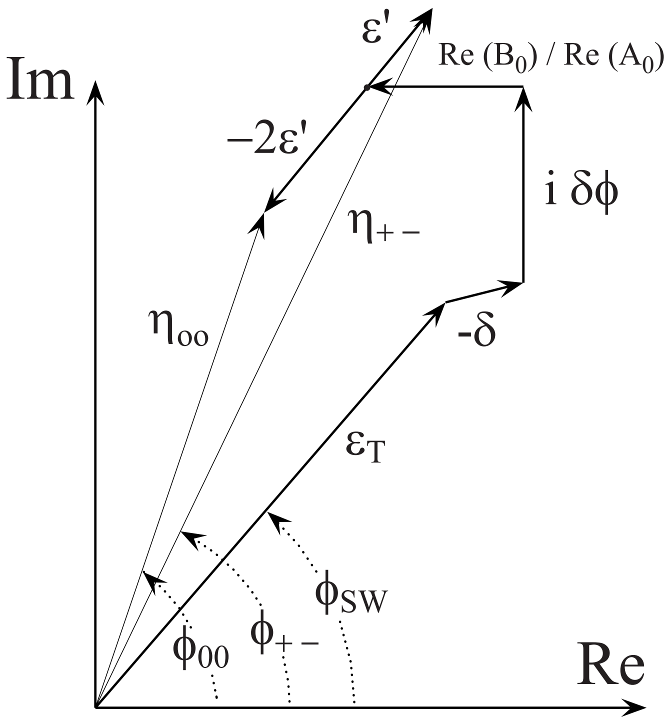

Figure 1: *CP*- and *CPT*-violation parameters in 2π decay.

$\delta_\perp$ can be obtained from the measurement of the $\pi\pi$ decays *CP*-violation parameters η_{+-} and η_{00}. Figure 1 shows the various contributions to $\eta_{\pi\pi}$ [1]. The T-violation parameter ϵ_T

$$\epsilon_T = i\frac{|\Lambda_{K^0\overline{K}^0}|^2 - |\Lambda_{\overline{K}^0K^0}|^2}{\Delta\gamma(\lambda_L - \lambda_S)} \tag{6}$$

has been defined in such a way that it is exactly aligned along the superweak direction [‡]. A_I (resp. B_I) is the CPT-conserving (resp. violating) decay amplitude for the $\pi\pi$ Isospin I state, ε' is the direct CP/CPT-violation parameter $[\varepsilon' = 1/3(\eta_{+-} - \eta_{00})]$ and $\delta\phi = \frac{1}{2}\left[\varphi_\Gamma - \arg(A_0^*\overline{A}_0)\right]$ is the phase difference between the $I = 0$ component of the decay amplitude and the matrix element $\Gamma_{K^0\overline{K}^0}$. From Fig. 1 one obtains

$$\begin{aligned}\delta_\perp =& |\eta_{+-}|(\phi_{SW} - \frac{2}{3}\phi_{+-} - \frac{1}{3}\phi_{00}) \\ & - \frac{\mathrm{Re}(B_0)}{\mathrm{Re}(A_0)}\sin(\phi_{SW}) + \delta\phi \ \ \cos(\phi_{SW}) \ . \end{aligned} \tag{7}$$

The present accuracy on the term $|\eta_{+-}|(\phi_{SW} - \frac{2}{3}\phi_{+-} - \frac{1}{3}\phi_{00})$ is 2.6×10^{-5}. $\delta\phi$ gets contributions from CP violation in semileptonic and 3π decays [2,3] and can only be neglected at the present time if one assumes that η_{000} is not significantly larger than η_{+-0}. Furthermore, B_0 is not directly measured, so additional assumptions (for example, CPT conservation in the decay which implies $B_0 = 0$) or a combination with other measurements are necessary to obtain $\delta_\perp$.

If one assumes unitarity, one can measure $\mathrm{Im}(\delta)$ using the Bell-Steinberger relation which relates K_S and K_L decay amplitudes into all final states f:

$$\mathrm{Re}(\epsilon_T) - i\mathrm{Im}(\delta) = \frac{1}{2(i\Delta m + \frac{1}{2}(\gamma_L + \gamma_S))} \times \sum A_{f_L} A^*_{f_S} \ . \tag{8}$$

Since the $\pi\pi$ amplitudes dominate, the result relies also strongly on the $\phi_{\pi\pi}$ phase measurements. The advantage is that B_0 does not enter. Using all available data, one obtains a value of $\mathrm{Im}(\delta)$ compatible with zero with a precision of 2×10^{-5}. The precision here is limited by the measurement of η_{+-}.

The results on $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$ can be combined to obtain $\delta_\parallel$ and $\delta_\perp$ and therefore the K^0–$\overline{K}^0$ mass and width difference shown in Fig. 2. The current accuracy is a few 10^{-18} GeV for both.

If one assumes that CPT is conserved in the decays ($\gamma_{K^0} = \gamma_{\overline{K}^0}$, $\delta_\parallel = 0$, $B_I = 0$), the phase of δ is known, and the $\delta_\perp$ and

Bell-Steinberger methods are identical. One in this case obtains a limit for $|m_{K^0} - m_{\overline{K}^0}|$ of 4.7×10^{-19} GeV (90%CL).

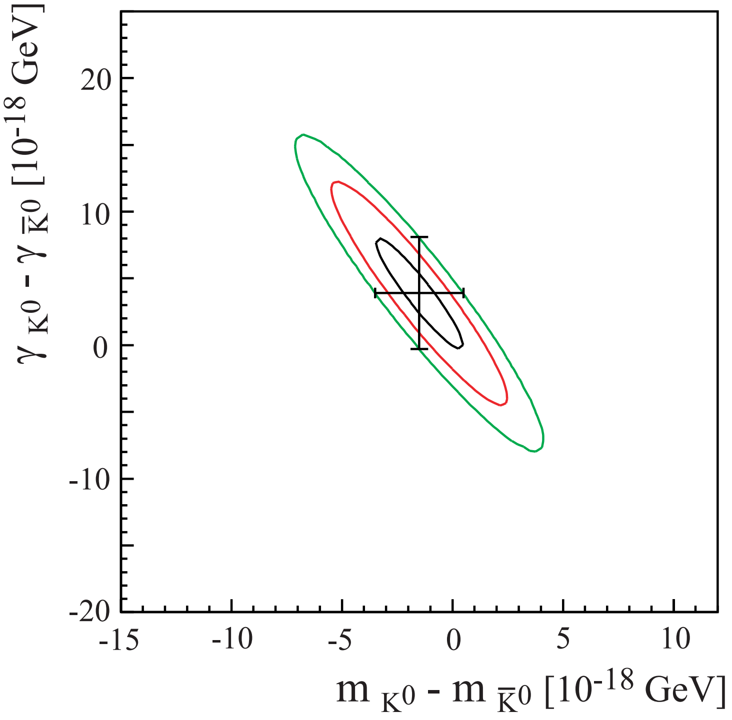

Figure 2: K^0–$\overline{K}^0$ mass vs width difference.

Footnotes and References

[‡] Many authors have a different definition of the T-violation parameter, $\epsilon = (\Lambda_{\overline{K}^0 K^0} - \Lambda_{K^0\overline{K}^0})/(2(\lambda_L - \lambda_S))$. ϵ is not exactly aligned with the superweak direction. The two definitions can be related through $\epsilon = \epsilon_T + i\delta\phi$.

1. See for instance, C.D. Buchanan *et al.*, Phys. Rev. **D45**, 4088 (1992). See also the Second Daphne Handbook, Ed. L.Maiani *et al.*, INFN Frascati (1995).
2. V.V. Barmin *et al.*, Nucl. Phys. **B247**, 293 (1984).
3. L. Lavoura, Mod. Phys. Lett. **A7**, 1367 (1992).

CP-VIOLATION PARAMETERS

Re(ϵ)

VALUE (units 10^{-3})	*DOCUMENT ID*	*TECN*
1.664±0.010	[2] LAI 05A	NA48

[2] LAI 05A values are obtained through unitarity (Bell-Steinberger relations), improving determination of η_{000} and combining other data from PDG and APOSTOLAKIS 99B.

CPT-VIOLATION PARAMETERS

In K^0-$\overline{K}^0$ mixing, if *CP*-violating interactions include a *T* conserving part then

$$|K_S\rangle = [|K_1\rangle+(\epsilon+\delta)|K_2\rangle]/\sqrt{1+|\epsilon+\delta|^2}$$
$$|K_L\rangle = [|K_2\rangle+(\epsilon-\delta)|K_1\rangle]/\sqrt{1+|\epsilon-\delta|^2}$$

where

$$|K_1\rangle = [|K^0\rangle + |\overline{K}^0\rangle]/\sqrt{2}$$
$$|K_2\rangle = [|K^0\rangle - |\overline{K}^0\rangle]/\sqrt{2}$$

and

$$|\overline{K}^0\rangle = CP|K^0\rangle.$$

The parameter δ specifies the *CPT*-violating part.

Estimates of δ are given below assuming the validity of the ΔS=ΔQ rule. See also THOMSON 95 for a test of *CPT*-symmetry conservation in K^0 decays using the Bell-Steinberger relation.

REAL PART OF δ

A nonzero value violates *CPT* invariance.

VALUE (units 10^{-4})	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
2.9± 2.6±0.6	1.3M	[3] ANGELOPO... 98F	CPLR	
• • • We do not use the following data for averages, fits, limits, etc. • • •				
2.4± 2.8		[4] APOSTOLA... 99B	RVUE	
180 ±200	6481	[5] DEMIDOV 95		$K_{\ell 3}$ reanalysis

[3] If ΔS=ΔQ is not assumed, ANGELOPOULOS 98F finds Reδ=$(3.0 \pm 3.3 \pm 0.6) \times 10^{-4}$.

[4] APOSTOLAKIS 99B assumes only unitarity and combines CPLEAR and other results.

[5] DEMIDOV 95 reanalyzes data from HART 73 and NIEBERGALL 74.

IMAGINARY PART OF δ

A nonzero value violates *CPT* invariance.

VALUE (units 10^{-5})	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
− 0.2± 2.0		[6] LAI 05A	NA48	
• • • We do not use the following data for averages, fits, limits, etc. • • •				
2.4± 5.0		[7] APOSTOLA... 99B	RVUE	
− 90 ± 290 ±100	1.3M	[8] ANGELOPO... 98F	CPLR	
2100 ±3700	6481	[9] DEMIDOV 95		$K_{\ell 3}$ reanalysis

[6] LAI 05A values are obtained through unitarity (Bell-Steinberger relations), improving determination of η_{000} and combining other data from PDG and APOSTOLAKIS 99B.

[7] APOSTOLAKIS 99B assumes only unitarity and combines CPLEAR and other results.

[8] If ΔS=ΔQ is not assumed, ANGELOPOULOS 98F finds Imδ=$(-15 \pm 23 \pm 3) \times 10^{-3}$.

[9] DEMIDOV 95 reanalyzes data from HART 73 and NIEBERGALL 74.

Re(y)

A non-zero value would violate *CPT* invariance in $\Delta S = \Delta Q$ amplitude. Re(y) is the following combination of K_{e3} decay amplitudes:

$$\mathrm{Re(y)} = \mathrm{Re}\Big(\frac{A(\overline{K}^0 \rightarrow e^- \pi^+ \overline{\nu}_e)^* - A(K^0 \rightarrow e^+ \pi^- \nu_e)}{A(\overline{K}^0 \rightarrow e^- \pi^+ \overline{\nu}_e)^* + A(K^0 \rightarrow e^+ \pi^- \nu_e)} \Big)$$

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*	*TECN*
0.4±2.5	13k	[10] AMBROSINO 06E	KLOE
• • • We do not use the following data for averages, fits, limits, etc. • • •			
0.3±3.1		[11] APOSTOLA... 99B	CPLR

[10] They use the PDG 04 (web update) for the K^0_L semileptonic charge asymmetry and PDG 04 (*CP* review, *CPT* NOT ASSUMED) for Re(ϵ).

[11] Constrained by Bell-Steinberger (or unitarity) relation.

Re(x_)

A non-zero value would violate *CPT* invariance in decay amplitudes with $\Delta S \neq \Delta Q$. x_, used here to define Re(x_), and x_+, used below in the $\Delta S = \Delta Q$ section are the following combinations of K_{e3} decay amplitudes:

$$x_{\pm} = \frac{1}{2}\Big(\frac{A(\overline{K}^0 \rightarrow \pi^- e^+ \nu_e)}{A(K^0 \rightarrow \pi^- e^+ \nu_e)} \pm \frac{A(K^0 \rightarrow \pi^+ e^- \overline{\nu}_e)^*}{A(\overline{K}^0 \rightarrow \pi^+ e^- \overline{\nu}_e)^*} \Big).$$

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
−0.8± 2.5	13k	[12] AMBROSINO 06E	KLOE	Tagged K^0_S
• • • We do not use the following data for averages, fits, limits, etc. • • •				
−0.5± 3.0		[13] APOSTOLA... 99B	CPLR	Strangeness tagged
2 ±13 ±3	650k	ANGELOPO... 98F	CPLR	Strangeness tagged

[12] Uses PDG 04 (web update) for the K^0_L semileptonic charge asymmetry and Re(δ) from CPLEAR, ANGELOPOULOS 98F.

[13] Constrained by Bell-Steinberger (or unitarity) relation.

$\left| m_{K^0} - m_{\overline{K}^0} \right| / m_{\text{average}}$

A test of *CPT* invariance. "Our Evaluation" is described in the "Tests of Conservation Laws" section. It assumes *CPT* invariance in the decay and neglects some contributions from decay channels other than $\pi\pi$.

VALUE	*CL%*	*DOCUMENT ID*	*TECN*
< 10^{-18} (CL = 90%) OUR EVALUATION			
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$(-\ 3\pm4) \times 10^{-18}$		[14] ANGELOPO... 99B	RVUE

[14] ANGELOPOULOS 99B assumes only unitarity and combines CPLEAR and other results.

$(\Gamma_{K^0} - \Gamma_{\overline{K}^0})/m_{\text{average}}$

A test of *CPT* invariance.

VALUE	*DOCUMENT ID*	*TECN*
$(7.8\pm8.4) \times 10^{-18}$	[15] ANGELOPO... 99B	RVUE

[15] ANGELOPOULOS 99B assumes only unitarity and combines CPLEAR with other results. Correlated with $(m_{K^0} - m_{\overline{K}^0})\ /\ m_{\text{average}}$ with a correlation coefficient of −0.95.

TESTS OF $\Delta S = \Delta Q$ RULE

Re(x_+)

A non-zero value would violate the $\Delta S = \Delta Q$ rule in *CPT* conserving transitions. x_+ is defined above in the Re(x_-) section.

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*		*TECN*
-0.8 ± 3.1 OUR AVERAGE				
-0.5 ± 3.6	13k	[16] AMBROSINO	06E	KLOE
-1.8 ± 6.1		[17] ANGELOPO...	98D	CPLR

[16] Re(x_+) can be shown to be equal to the following combination of rates:

$$\mathrm{Re}(x_+) = \frac{1}{2}\,\frac{\Gamma(K^0_S \to \pi e \nu) - \Gamma(K^0_L \to \pi e \nu)}{\Gamma(K^0_S \to \pi e \nu) + \Gamma(K^0_L \to \pi e \nu)}$$

which is valid up to first order in terms violating *CPT* and/or the $\Delta S = \Delta Q$ rule.

[17] Obtained neglecting *CPT* violating amplitudes.

K^0 REFERENCES

ABOUZAID	06	PRL 96 101801	E. Abouzaid *et al.*	(KTEV Collab.)
AMBROSINO	06E	PL B636 173	F. Ambrosino *et al.*	(KLOE Collab.)
LAI	05A	PL B610 165	A. Lai *et al.*	(CERN NA48 Collab.)
PDG	04	PL B592 1	S. Eidelman *et al.*	
LAI	03C	EPJ C30 33	A. Lai *et al.*	(CERN NA48 Collab.)
LAI	02	PL B533 196	A. Lai *et al.*	(CERN NA48 Collab.)
ANGELOPO...	01B	EPJ C22 55	A. Angelopoulos *et al.*	(CPLEAR Collab.)
ANGELOPO...	99B	PL B471 332	A. Angelopoulos *et al.*	(CPLEAR Collab.)
APOSTOLA...	99B	PL B456 297	A. Apostolakis *et al.*	(CPLEAR Collab.)
ANGELOPO...	98D	PL B444 38	A. Angelopoulos *et al.*	(CPLEAR Collab.)
Also		EPJ C22 55	A. Angelopoulos *et al.*	(CPLEAR Collab.)
ANGELOPO...	98E	PL B444 43	A. Angelopoulos *et al.*	(CPLEAR Collab.)
ANGELOPO...	98F	PL B444 52	A. Angelopoulos *et al.*	(CPLEAR Collab.)
Also		EPJ C22 55	A. Angelopoulos *et al.*	(CPLEAR Collab.)
DEMIDOV	95	PAN 58 968 From YAF 58 1041.	V. Demidov, K. Gusev, E. Shabalin	(ITEP)
THOMSON	95	PR D51 1412	G.B. Thomson, Y. Zou	(RUTG)
BARKOV	87B	SJNP 46 630 Translated from YAF 46 1088.	L.M. Barkov *et al.*	(NOVO)
BARKOV	85B	JETPL 42 138 Translated from ZETFP 42 113.	L.M. Barkov *et al.*	(NOVO)
BLATNIK	79	LNC 24 39	S. Blatnik, J. Stahov, C.B. Lang	(TUZL, GRAZ)
MOLZON	78	PRL 41 1213	W.R. Molzon *et al.*	(EFI+)
NIEBERGALL	74	PL 49B 103	F. Niebergall *et al.*	(CERN, ORSAY, VIEN)
HART	73	NP B66 317	J.C. Hart *et al.*	(CAVE, RHEL)
FOETH	69B	PL 30B 276	H. Foeth *et al.*	(AACH, CERN, TORI)
HILL	68B	PR 168 1534	D.G. Hill *et al.*	(BNL, CMU)
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CHRISTENS...	64	PRL 13 138	J.H. Christenson *et al.*	(PRIN)
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ROSENFELD	59	PRL 2 110	A.H. Rosenfeld, F.T. Solmitz, R.D. Tripp	(LRL)