Economic planning, computers and labor values

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Abstract

Since the collapse of the Soviet Union it has seemed to many that the socialist calculation debate is essentially over, with a decisive verdict in favor of the market. Recent instabilities in the world market are again prompting the question whether some form of conscious regulation of economies may be appropriate. We argue that the increasing power of modern computer technology along with the use of Ricardian–Marxian labor values opens up new possibilities for economic planning.

1 Introduction

The collapse of the Soviet Union at the end of the 1980s has established a strong presumption—reinforced by the arguments of the Austrian school (Hayek, Mises)—that there exists no viable alternative to capitalism and the free market. From this perspective, socialist planning appears as a utopian dream. Not only have socialists made very few attempts to defend planning of late; there has been very little substantive discussion of economic planning at all. One index of the dominance of the Austrian arguments regarding the impossibility of rational planning is provided by Joseph Stiglitz's *Whither Socialism* (1994). Stiglitz is critical of socialist economics, but his critique is almost entirely directed against *market* socialism. As for a centrally planned economy, he says only that "Hayek had rightly criticized" the Marxian project, "arguing that the central planner could never have the requisite information" (Stiglitz, 1994, p. 9). This is a typical response: even economists who do not subscribe fully to Hayek's views on the merits of the free market nonetheless generally believe that the Austrian critique of central planning may safely be regarded as definitive. We hope to show that this should not be taken for granted.

The next section outlines our proposals for a system of rational socialist planning; section 3 assesses the technical feasibility of implementing these proposals. The scheme we advocate involves making extensive use of labor values (in the sense of vertically integrated labor coefficients) in the planning process, and in section 4 we examine the criticism of this sort of use of labor values put forward by Samuelson and Weizsäcker. Section 5 extends this argument, drawing on empirical work which suggests that the "bourgeois prices" (or in Marxian terminology, prices of production) favored by Samuelson and Weizsäcker for economic calculation are not generally to be found in capitalist economies. A brief conclusion is presented in section 6.

2 Outline of our proposals

We first set out the general conditions which are required to operate an effective system of central economic planning, leaving aside for the moment the issue of whether they can be realized in any feasible system. Taking an input–output perspective on the economy, effective central planning requires the following basic elements:

1. A system for arriving at (and periodically revising) a set of targets for final outputs, which incorporates information on both consumers' preferences and the relative cost of producing alternative goods (the appropriate metric for cost being left open for the moment).

2. A method of calculating the implications of any given set of final outputs for the required gross outputs of each product. At this stage there must also be a means of checking the feasibility of the resulting set of gross output targets, in the light of the constraints posed by labor supply and existing stocks of fixed means of production, before these targets are forwarded to the units of production.

The provision of these elements involves certain preconditions, notably an adequate system for gathering and processing dispersed economic information and a rational metric for cost of production. We should also note the point stressed by Nove (1977 and 1983): for effective central planning, it is necessary that the planners are able to carry out the above sorts of calculations in full disaggregated detail. In the absence of horizontal market links between enterprises, management at the enterprise

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level "*cannot* know what it is that society needs unless the centre informs it" (Nove, 1977: 86).[1] Thus if the centre is unable to specify a coherent plan in sufficient detail, the fact that the plan may be balanced in aggregate terms is of little avail. Even with the best will in the world on the part of all concerned, there is no guarantee that the specific output decisions made at the enterprise level will mesh properly. This general point is confirmed by Yun (1988: 55), who states that as of the mid-1980s Gosplan was able to draw up material balances for only 2,000 goods in its annual plans. When the calculations of Gossnab and the industrial ministries are included, the number of products tracked rises to around 200,000, still far short of the 24 million items produced in the Soviet economy at the time. This discrepancy meant that it was "possible for enterprises to fulfill their plans as regards the nomenclature of items they have been directed to produce, failing at the same time to create products immediately needed by specific users".

Our argument involves grasping this nettle: while we agree that "in a basically non-market model the centre must discover what needs doing" (Nove, 1977: 86), and we accept Yun's account of the failure of Gosplan to do so, we dispute Nove's contention that "the centre cannot do this in micro detail" (*ibid.*).

Our basic proposals can be laid out quite simply, although we ask the reader to bear in mind that we do not have space here for the necessary refinements, qualifications and elaborations (these are developed at length in Cockshott and Cottrell, 1993). In schematic form the proposals are as follows.

2.1 *Labor time as social unit of account and measure of cost*

The allocation of resources to the various spheres of productive activity takes the form of a social labor budget. At the same time the principle of labor time minimization is adopted as the basic efficiency criterion. We are in agreement with Mises (1935: 116) that rational socialist calculation requires "an objectively recognizable unit of value, which would permit of economic calculation in an economy where neither money nor exchange were present. And only labour can conceivably be considered as such." We disagree with Mises' subsequent claim that even labor time cannot, after all, play the role of objective unit of value. We have countered his two arguments to this effect—namely, that labor-time calculation necessarily leads to the undervaluation of non-reproducible natural resources, and that there is no rational way (other than via a system of market-determined wage rates) of reducing labor of differing skill levels to a common denominator—in another publication (Cottrell and Cockshott, 1993a). We can only summarize out responses here. If one uses *marginal* labor time as a measure of cost, that takes into account the growing difficulty in obtaining non-reproducible resources. In addition, planners could decide to devote resources to the research into alternatives, the use of solar power instead of oil for instance. Furthermore, there is no reason to believe that any real market furnishes an optimal solution to such problems. As for the non-homogeneity of labor, one can in principle treat skilled labor in the same way as any other product, evaluated in terms of the training time required to produce it.

2.2 *Labor-token system of distribution*

From Marx's *Critique of the Gotha Programme* (Marx, 1974) we take the idea of the payment of labor in "labor tokens", and the notion that consumers may withdraw from the social fund goods having a labor content equal to their labor contribution (after deduction of taxes to offset the communal uses of labor time: accumulation of means of production, public goods and services, support of those unable to work). We envisage a basically egalitarian pay system; but insofar as departures from egalitarianism are made (i.e. some kinds of work are rewarded at more than, and some at less than, one token per hour), the achievement of macroeconomic balance nonetheless requires that the total current issue of labor tokens equals the total current labor performed. We also suggest that the most suitable system of taxation in such a context is a flat tax per worker—a uniform membership fee for socialist society, so to speak. This tax (net of transfers to non-workers) should, in effect, cancel just enough of the current issue of labor tokens so as to leave consumers with sufficient disposable tokens to purchase the output of consumer goods at par. (This point is further developed below.)

2.3 *Democratic decisions on major allocation questions*

The allocation of social labor to the broad categories of final use (accumulation of means of production, collective consumption, personal consumption) is suitable material for democratic decision making. This might take various forms: direct voting on specific expenditure categories at suitable intervals (e.g. on whether to increase, reduce or maintain the proportion of social labor devoted to the health care system), voting on a number of pre-balanced plan variants, or electoral competition between

[1] With one reservation. If, say, the central plan calls for enterprise A to supply intermediate good x to enterprise B, where it will be used in the production of some further good y, and if the planners apprise A and B of this fact, is there not scope for "horizontal" discussion between the two enterprises over the precise design specification of x, even in the absence of market relations between A and B?

parties with distinct platforms as regards planning priorities.

2.4 Consumer goods algorithm

Our proposal on this count may be described as "Lange plus Strumilin". From Lange (1938) we take up a modified version of the trial and error process, whereby market prices for consumer goods are used to guide the allocation of social labor among the various consumer goods; from Strumilin we take the idea that in socialist equilibrium the use-value created in each line of production should be in a common proportion to the social labor time expended.[2]

The central idea is this: the plan calls for production of some specific vector of final consumer goods, and these goods are marked with their social labor content. If planned supplies and consumer demands for the individual goods happen to coincide when the goods are priced in accordance with their labor values,[3] the system is already in equilibrium. In a dynamic economy, however, this is unlikely. If supplies and demands are unequal, the marketing authority for consumer goods is charged with adjusting prices, with the aim of achieving (approximate) short-run balance, i.e. prices of goods in short supply are raised while prices are lowered in the case of surpluses.[4] In the next step of the process, the planners examine the ratios of market-clearing price to labor value across the various consumer goods. (Note that both of these magnitudes are denominated in labor-hours; labor content in the one case, and labor tokens in the other). Following Strumilin's conception, these ratios should be equal (and equal to unity) in long-run equilibrium. The consumer goods plan for the next period should therefore call for expanded output of those goods with an above-average price/value ratio, and reduced output for those with a below-average ratio.[5]

In each period, the plan should be balanced, using either input–output methods or an alternative balancing algorithm.[6] That is, the gross outputs needed to support the target vector of final outputs should be calculated in advance. This is in contrast to Lange's (1938) system, in which the very coherence of the plan—and not only its optimality—seems to be left to trial and error. Our scheme, however, does not impose the requirement that the pattern of consumer demand be perfectly anticipated *ex ante*; adjustment in this respect is left to an iterative process which takes place in historical time.[7]

The proposed scheme as a whole is set out in synoptic form in Figure 1.

This scheme meets the objection of Nove (1983), namely that labor values cannot provide a basis for planning even if they gave a valid measure of cost of production. Nove's point is that labor content of itself tells us nothing about the use-value of different goods. Of course this is true,[8] but it only means that we need an independent measure of consumers' valuations; and the price, in labor tokens, which roughly balances planned supply and consumer demand provides just such a measure. By the same token, we can answer a point made by Mises in his discussion of the problems faced by socialism under dynamic conditions (1951: 196ff). One of the dynamic factors he considers is change in consumer demand, à propos of which he writes: "If economic calculation and therewith even an approximate ascertainment of the costs of production were possible, then within the limits of the total consumption-units assigned to him, each individual citizen could be allowed to demand what he liked. . . ." But, he continues, "since, under socialism, no such calculations are possible, all such questions of demand must necessarily be left to the government". Our proposal allows for precisely the consumer choice that Mises claims is unavailable.

3 Feasibility of calculation

3.1 Calculation of labor values

The proposals above rest on the assumption that it is possible to calculate the labor content of each product in the economy. The problem is in principle solvable since one has n unknown labor values related by a set of n linear production functions. The difficulty is not one of princi-

[2] This point—a basic theme of Strumilin's work over half a century—is expressed particularly clearly in his (1977: 136–7).

[3] A point of terminology should be noted here. We use the term "labor value", or just "value" in some contexts, to mean the sum of the labor time directly and indirectly required to produce a given product, or in other words the product's vertically integrated labor coefficient. We take this usage to be basically in line with that of Ricardo and Marx, but some Marx scholars would not agree—cf. Freeman and Carchedi (1996).

[4] With market-clearing prices, of course, the goods go to those willing to pay the most. Given an egalitarian distribution of income, we see no objection to this.

[5] Naturally, an element of demand forecasting is called for here: the current ratios provide a useful guide rather than a mechanical rule.

[6] An alternative algorithm which makes allowance for given stocks of specific means of production is given in Cockshott (1990). This algorithm is discussed in section 3.3 below.

[7] In his later reflection on the socialist calculation debate, Lange (1967) seems to suggest that an optimal plan can be pre-calculated by computer, without the need for the real-time trial and error he envisaged in 1938. Insofar as this would require that consumer demand functions are all known in advance, this seems to us far-fetched.

[8] As was clearly understood by Marx: "On a given basis of labour productivity the production of a certain quantity of articles in every particular sphere of production requires a definite quantity of social labour-time; although this proportion varies in different spheres of production and has no inner relation to the usefulness of these articles or the special nature of their use-values (1972: 186–7)."

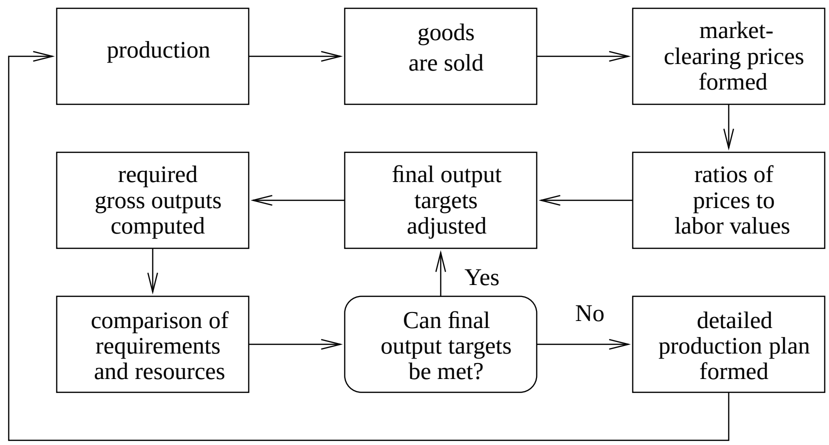

Figure 1: Outline of planning mechanism

ple but of scale.[9] When the number of products gets up into the millions, the calculation involved is nontrivial.

If we represent the problem in the standard form, via an n by $(n+1)$ matrix where the rows represent products and the columns represent produced inputs plus direct labor, analytic solution of the equations using Gaussian elimination requires n^3 multiplication operations and a slightly larger number of additions and subtractions. Table 1 shows the computer requirements for this calculation assuming differing sizes of economy. We assume that the uniprocessor is capable of 10^8 multiplications a second and that the multiprocessor can perform 10^{10} multiplications per second.

It can be seen that, taking compute time alone into account, even the multiprocessor would take 10^{11} seconds, or over three thousand years, to produce a solution for an economy of 10 million products. The situation is further complicated by the memory required to store the matrix, which grows as n^2. Since the largest currently feasible memories are of the order of 10^{10} words this would set a limit on the size of problem that could be handled at about 100,000 products.

If, however, we take into account the *sparseness* of the matrix (i.e. the high proportion of zero entries, when it is specified in full detail) the problem becomes more tractable. Let us suppose that the number of different types of components that enter directly into the production of any single product is n^k where $0 < k < 1$. If we assume a value of 0.4 for k, which seems fairly conservative,[10] we find that memory requirements now grow as $n^{(1+k)} = n^{1.4}$. If we can further simplify the problem by using iterative numerical techniques (Gauss–Seidel or Jacobi, see Varga, 1962) to obtain approximate solutions, we obtain a computational complexity function of order $An^{1.4}$, where A is a small constant determined by the accuracy required of the answer.

This reduces the problem to one that is clearly within the scope of current computer technology, as shown in Table 2. The most testing requirement remains the memory, but it is within the range of currently available machines.

We conclude that the computation of labor values is eminently feasible.

3.2 Resource Allocation

If we assume that the mix of net or final outputs required by the plan is specified, as are the available technologies and the stocks of means of production, how difficult is it to compute a feasible plan? (By a "feasible" plan we mean one which produces at least the required outputs using the available resources.) Following on from this, can we determine if the planned mix of outputs is infeasible given the resources?

The classic approach to this problem involves the use of linear programming, whose computational requirements are unfortunately forbidding for an economy with millions of products. But if we are willing to relax our requirements somewhat and settle for a "good" rather than an optimal solution, we can perform a simplification similar to that described for labor-value calculations. One approach would be to start from the target

[9] There is a difficulty of principle in the case of joint production. But we have shown (Cottrell and Cockshott, 1993a, appendix) that this is not a serious obstacle to the use of labor values in the planning process, as described above. Basically, it is enough to be able to compare the aggregate price and the aggregate value of the specific goods that are jointly produced.

[10] This means, for instance, that in a 10 million product economy each product is assumed to have on average 631 direct inputs.

Number of products	Multiplications	Time taken in seconds: Uniprocessor	Multiprocessor
1,000	1,000,000,000	10	0.1
100,000	10^{15}	10^{7}	100,000
10,000,000	10^{21}	10^{13}	10^{11}

Table 1: Gaussian solution to labor values

Number of products	Multiplic-ations	Words of memory	Time taken in seconds: Uniprocessor	Multiprocessor
1,000	158,489	31,698	1.6×10^{-3}	1.6×10^{-5}
100,000	100,000,000	20,000,000	1	0.01
10,000,000	6.3×10^{10}	1.2×10^{10}	630	6.3

Table 2: Iterative solution to labor values (Assuming A=10)

list of final outputs, and work back to the corresponding required gross outputs (via the same sort of iterative solution method set out for labor values, and exploiting the sparseness of the input–output matrix in the same way). Given the vector of gross outputs, it is then straightforward to determine the overall requirements for labor and fixed means of production of various kinds. If the latter requirements can be met, well and good; and if not, one trims the target list of final outputs and tries again. These steps are shown in the form of a loop at the bottom left of Figure 1. While it is computationally feasible, this method has the drawbacks of requiring a manual adjustment of the target output vector each time round the loop, and of failing to ensure that all resources are used as fully as possible.

3.3 *Low complexity plan balancing*

A preferable alternative technique, which draws on ideas from the literature on neural nets, is set out in Cockshott (1990). This is of complexity $An^{(1+k)}$, as was the iterative solution for labor values. The computational requirements are thus essentially the same. The procedure involves defining a metric for the degree of fit between the target set of final outputs and the computed feasible set, as constrained by existing stocks of means of production of various kinds, and by the available labor time.

The problem is to combine a set of technologies, stocks and flows of goods in such a way as to best meet a set of target outputs g_i. We let the subscript i range over outputs, and the subscript j range over stocks and flows. Thus S_{ij} stands for the stock of good j used in the production of output i and F_{ij} denotes the flow of good j used in the production. We assume that stocks and flows take on integer values (i.e. goods are not infinitely divisible). We assume also that there is a linear relationship between the output of a product and the stocks and flows required to produce it.

$$S_{ij} = O_i c_j \tag{1}$$

$$F_{ij} = O_i f_j \tag{2}$$

where O_i is the output of the ith product and c_j, f_j are technology specific constants.

The algorithm draws on techniques developed in simulated neural networks, in particular upon the notion of a “harmony function” (Smolensky, 1986). We define a real valued harmony function $H(g, O) = 1 - \frac{1}{(O_i/g_i)}$ (see Figure 2) which takes on negative values when output falls below the goal and a positive value when it exceeds the goal. Marginal harmony is a declining function of output, which encodes the notion that deficits are a more serious problem than surpluses are a benefit.

For our simulated model of the economy we start off with an arbitrary initial allocation of resources. In a real planning context one could start off with the actually existing allocation of resources between ingustries.

The algorithm then is as follows:

1. For each industry determine the level of output that can be obtained with the current resource allocation, O_i^0.
2. For each industry determine which input stock acts as the rate limiting factor for production.
3. Given a linear production function we can then determine how much of each other input each industry requires in order to sustain a gross output of O_i^0.

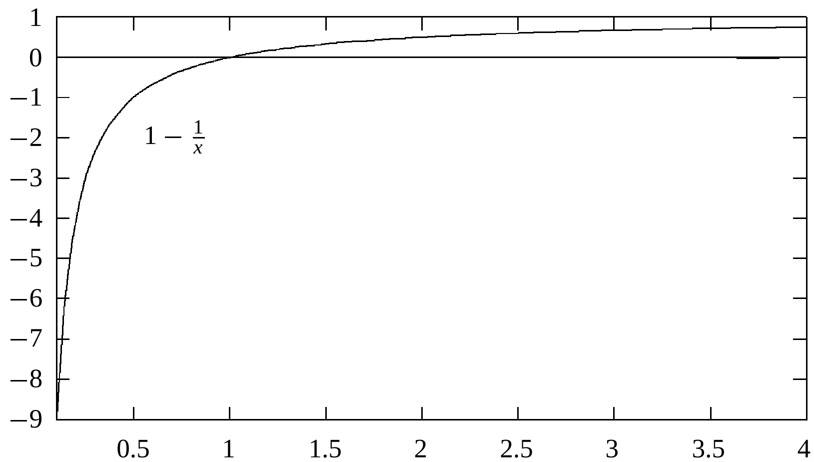

Figure 2: The harmony function $(1 - \frac{1}{(O_i/g_i)})$

We call these stocks the balancing stocks and denote them by $b^0_{i,j}$. Given the balancing stocks we deduct these from the initial allocated stocks, and logically allocate the residual stocks to a global reserve.

4. Compute the net product of each good across the economy as a whole, and thus the harmony of each industry.
5. Compute the mean harmony for the whole economy.
6. Sort the industries in order of their harmony.
7. For each industry starting with the least harmonious:
   (a) If sufficient stocks are available in the global pool allocate enough of each good to the industry to bring its production level up to a point at which its harmony would equal the mean harmony of the whole economy.
   (b) If insufficient stocks are available to do this, allocate as much as is available.
8. All steps up to now have either increased or conserved harmony of each product. We now reallocate resources from those industries with the highest harmony to industries where harmony is low, and the partial derivative of the harmony function is high with respect to the given input. This can be done by
   (a) computing, for each product, the mean of the partial derivatives of all the harmony functions of the industries for which it appears as an input;
   (b) taking sufficient stocks away from those industries in the top decile to reduce the output of these industries to the mean harmony; and
   (c) allocating the resulting stocks to the global pool.
9. Iterate steps 2 and 8 until the increase in overall harmony is smaller than some pre-given constant ϵ.

The complexity of the algorithm is of order $RkNM$ where R is the number of iterations, N the number of products and M the mean number of inputs per industry. The accuracy of the result, measured in digits of deviation of total harmony from its limit value, grows linearly with R. The expected value of R is thus small compared to N and M, and in general we have the relation $N \gg M > R$. The overall compute time is thus of the same order as finding labour values.

The algorithm above searches the space of feasible plans, aiming to maximize the degree of fit between feasible output and target output. The nature of the search algorithm is such that it may settle at a local maximum rather than finding the global maximum; this is the price paid for computational tractability. Nonetheless, that the solution is not the optimal plan, but merely a good feasible one, is not a serious problem when comparing planning to the market, since no real market achieves an optimal structure of production.

3.4 Comparison with existing computer technology

We have set out the scale of computer resources required to compute labor values and to compute a feasible plan for a whole economy. The required memory and processing power are well within the capabilities of current machines. We took as our benchmark a multiprocessor capable of 10^{10} multiplications a second; the peak rates of machines in use in research institutions today exceed 10^{12} operations per second.[11] One must allow some reduction in peak rates before arriving at a sustainable performance for a computer, but our target performance is clearly realistic. Memory requirements are also within the range of current products. With modern computers, one could envisage computing an updated list of labor values daily and preparing a new perspective plan weekly—somewhat faster than a market economy is able to react.

4 The argument for "bourgeois pricing"

We have proposed using "simple" labor values as a measure of cost of production. But is it not well known that a rational planner could do better than this? Are we not condemning our economic calculus to sub-optimality by ignoring the dating of labor? This question arises in two contexts: our idea that planning should aim toward an equilibrium in which the labor-token prices of consumer goods are equalized with their labor values, and our proposal that choice of technique should be guided by the criterion of minimizing the required labor time.

We explore both of these issues below, taking as our starting point Samuelson and Weizsäcker's (1972) discussion of "rational planning through use of the bourgeois profit rate".[12] In the following section we consider the further, related question of whether pricing in actual capitalist economies conforms to Samuelson's "bourgeois" prices, in the sense of a uniform profit markup: we argue that it is not at all clear that this is the case.

4.1 Bourgeois prices in the planned economy

Samuelson and Weizsäcker set the scene for their argument by noting the way in which a positive rate of profit disturbs the simple labor theory of value:

> In an economic system where all goods are ultimately producible by labor… if the rate of profit or interest were always zero, the competitive equilibrium prices would be exactly equal to the total embodied labour required for each good.… If, however, there is a positive interest or profit rate, labor will not receive a real wage large enough to buy all the consumption goods producible by labor in the stationary synchronised equilibrium.… [W]ith positive interest the prices will no longer be proportional to the respective embodied labor contents. Thus, if the same historic labor total, say 1 labor, is needed for either a liter of grape juice or for a liter of wine, but for wine the labor is needed 2 time-units earlier rather than only one time-unit earlier as for grape juice, the ratio of wine price to grape juice price will not be $P_2/P_1 = 1/1$, but will instead vary with the profit rate per period r, being $P_2/P_1 = 1(1+r)^2/1(1+r) = (1+r)$… Thus grape juice and wine have equal "values" since they both involve unit labor inputs; but their bourgeois "prices" differ from the Marxian values because the former calculate labor requirements, dated by when they occur and carried forward at nefarious compound interest. (SW, p. 312)

They argue that in a rationally planned society, where class exploitation is abolished, all goods should be "valued" or priced at their "synchronised needed labor cost". Such rational plan prices will, in general, not be proportional to sums of undated labor content, but will be expressible in the manner of bourgeois prices, provided that an appropriate profit rate is used.

The essence of the Samuelson argument can be expressed in terms of our own approach, by considering the labor-token prices of given commodities which will succeed in clearing the market, given the number of labor-tokens currently being issued and spent. This concept appears to correspond precisely with Samuelson's synchronised needed labor cost. Although Samuelson carries through the argument with the full generality of matrix algebra, the basic idea can be understood by analyzing an economy producing a pure consumer good and no intermediate output.

Take for example the two consumer goods mentioned in the citation above, grape juice and wine, with the technologies as stated there (each requires a unit labor input but the grape juice, G, requires it one period in advance of consumption while the wine, W, requires it two periods in advance). To investigate the rational plan prices we perform the thought experiment of having the economy specialize entirely in each of these commodities in turn.

We use the following notation:

L_t = total labor supply at time t, equal to the number of labor tokens issued at that time (and spent, within the same period).

C_{jt} = quantity of commodity j available for consumption at time t, in physical units.

[11] See the website www.cray.com, also Dongarra, Meuer and Strohmaier (1997).

[12] In subsequent references, "Samuelson and Weizsäcker (1972)" will be abbreviated as "SW".

P_{jt} = market-clearing price of commodity j at time t, defined as L_t/C_{jt}. This price, which is expressed in labor-tokens per physical unit, balances the quantity of the commodity currently available against the total expenditure of tokens in the same period.

Let us now consider a rational pricing policy in the cases considered by Samuelson and Weizsäcker.

First Case: *Population and labor supply are growing at a compound percentage rate g (≡ γ − 1), while production technology is static.*

As of time t, given the unit labor requirements for each commodity, we have

$$C_{Gt} = L_{t-1}$$

$$C_{Wt} = L_{t-2}$$

$$P_{Gt} = L_t/C_{Gt} = L_t/L_{t-1} = \gamma$$

$$P_{Wt} = L_t/C_{Wt} = L_t/L_{t-2} = \gamma^2$$

and the "rational" price ratio is not 1:1, but rather $P_{Wt}/P_{Gt} = \gamma^2/\gamma = \gamma$.

Here the "rational prices" or synchronised needed labor costs are equal to the labor contents marked up at a compound rate of $\gamma = (1 + g)$. These prices diverge from simple labor values, but are equal, as Samuelson puts it, to "bourgeois prices", using a profit rate of g.

> Synchronised labor costs, as defined here, are seen to be interpretable as the ordinary embodied labor requirements for a fictitious system in which every. . . [input] coefficient of the actual system is blown up by the growth factor $(1+g)$. What is the rationale for this expansion?
>
> In each time interval the population is larger, and if we make the assumptions that:
>
> a) there is no saving,
>
> b) total income is equal to total labor expended,
>
> c) the length of the working week is unchanged,
>
> then it follows that the total expenditure of income in each time period will be greater than the labor hours used in production during the previous period. This will induce an inflation of prices above their values. (SW, p. 313)

Note that under conditions of declining population, or when there is a reduction in the working week, the quantity g will be negative, and hence the price of wine will be less than that of grape juice.

Second Case: *Population and labor supply are static, but labor-augmenting technical change is proceeding in such a way that the labor input requirement for each commodity is falling at a compound rate of b per period.*

This implies that starting out with a unit labor requirement at time 0, the requirement at time t is given by β^{-t} where $\beta \equiv (1 + b)$, and the quantity of output per unit labor input at t is β^t.

We then have:

$$C_{Gt} = L_{t-1}\beta^{t-1}$$

$$C_{Wt} = L_{t-2}\beta^{t-2}$$

Since in this case $L_t = L_{t-1} = L_{t-2} = \cdots = L$, we have

$$P_{Gt} = L/C_{Gt} = \frac{L}{L\beta^{t-1}} = \frac{1}{\beta^{t-1}} = \beta^{1-t}$$

$$P_{Wt} = L/C_{Wt} = \frac{L}{L\beta^{t-2}} = \frac{1}{\beta^{t-2}} = \beta^{2-t}$$

and the wine/juice price ratio is $P_{Wt}/P_{Gt} = \beta$.

As Samuelson points out, in this case the optimal prices are precisely equal to the historic embodied labor contents. Wine is more expensive than grape juice by the factor $\beta = (1 + b)$; correspondingly the wine currently available for consumption was produced (involved a labor requirement) at an earlier date, when the productivity of labor was lower.

The prices given above can also be retrieved by taking the labor requirement as of the current state of technique and marking it up at a "profit rate" of b. In the case of wine at time t the current technique labor requirement is β^{-t}, but as the labor was applied two periods ago this is marked up by the factor β^2, yielding

$$P_{Wt} = \beta^2\beta^{-t} = \frac{1}{\beta^{t-2}}$$

which agrees with the market-clearing labor-token price calculated above. Samuelson refers to this again as the bourgeois price, applying profit rate b.

Third Case: *This combines the two previous cases: growth in labor supply at rate g and technical progress at rate b.*

We then have:

$$C_{Gt} = L_{t-1}\beta^{t-1}$$

$$C_{Wt} = L_{t-2}\beta^{t-2}$$

$$P_{Gt} = L_t/L_{t-1}\beta^{t-1} = \gamma/\beta^{t-1}$$

$$P_{Wt} = L_t/L_{t-2}\beta^{t-2} = \gamma^2/\beta^{t-2}$$

and the relative price of wine is $P_{Wt}/P_{Gt} = \gamma\beta$.

For example, suppose we start out with static population and technology, and a wine/juice price ratio of 1:1. Now, if population starts growing at 2% while labor productivity starts advancing at 4% per period, the optimal price ratio of wine to grape juice will shift to (1.02×1.04):1. Wine should cost 6.08% more than grape juice.

Again, Samuelson points out that these rational prices are equivalent to prices derived by marking up labor contents as required by current technique, using profit rate R, where $(1 + R) = \gamma\beta$.

4.2 *Assessment*

What should we make of these arguments? Despite Samuelson and Weizsäcker's use of the term "bourgeois prices", their discussion is not very relevant to the debate over the relative merits of capitalism and socialism, as actual pricing in capitalist economies is far removed from the kind of rationality upon which they insists. The rate of profit is far from uniform, and the rate of interest is subject to irrational fluctuations.

Nonetheless, the arguments given above may be relevant to the procedures that should be followed by a rational planning authority. If the planning authorities have at their disposal all the input–output coefficients, and are using these to calculate labor values from direct labor requirements, then it would not be very difficult to recalculate modified values along the lines suggested by Samuelson, by first "blowing up" all the input coefficients by an appropriate factor. We have suggested that consumer goods ought to be marked with their actual labor content, but for the purposes of determining target prices—in order to apply the consumer goods algorithm[13]—there may be some merit in this alternative. At least one could carry out sensitivity analysis to see how much difference it would make to the workings of the consumer goods algorithm.

4.3 *Choice of technique*

With regard to choice of technique, we wish to argue (a) that there may be a case for modifying the calculation based on undated labor time, under certain conditions, but (b) that real bourgeois pricing (in actual capitalist economies) is likely to produce results that compare unfavorably with the application of simple labor time minimization via socialist planning.

To develop these points it may be useful to consider a simple illustration. Suppose we have two methods of digging a ditch: one technique uses equal quantities of direct labor and labor time embodied in means of production, the other saves on labor but at the cost of additional implements. For instance a contractor might employ 2 men with pneumatic drills to dig the ditch, or one man with an earth moving machine.

method	direct labor	indirect labor	Total time
old	100	100	200
new	50	125	175

In terms of labor-time accounting the new method is superior; it saves society 25 hours of labor. Costing in money terms is likely to give a different result. Suppose that an hour's labor adds a value of £7.50 to the product, while a laborer is paid £3.00 per hour (fairly realistic values for British industry in the late 1980s). In terms of money cost we obtain:

method	direct labor	indirect labor	total money cost
old	100×£3	100×£7.50	£1050.00
new	50×£3	125×£7.50	£1087.50

In monetary terms the old technique is cheaper. This is because the contractor pays only for part of the labor expended by his workers while he pays for the whole cost of the labor embodied in machines. From the standpoint of labor time minimization the bourgeois calculation appears socially irrational, though profitable.

Now suppose that direct labor is applied one period in advance of output, and indirect labor two periods in advance. Let us apply the Samuelsonian criterion, using a profit parameter R. Then the respective costs of the two methods are:

$$\begin{array}{ll} \text{old} & 100(1+R) + 100(1+R)^2 \\ \text{new} & 50(1+R) + 125(1+R)^2 \end{array}$$

and the condition for the social superiority of the new method is then

$$50(1+R) + 125(1+R)^2 < 100(1+R) + 100(1+R)^2$$

$$\Rightarrow 2(1+R) + 5(1+R)^2 < 4(1+R) + 4(1+R)^2$$

$$\Rightarrow (1+R)^2 < 2(1+R)$$

$$\Rightarrow (1+R) < 2 \Rightarrow R < 1$$

If the parameter R is less than 100% the new method is socially superior, otherwise the old method remains superior, on this criterion. For any set of comparative costs there would be a corresponding critical figure for R.

[13] See section 2.4 above, and for more detail Cottrell and Cockshott (1993), Chapter 8.

Now return to the capitalist calculation. To generalize it, let the wage rate, W, and the amount of value created per hour, H, be considered as variables. The monetary costs of the two methods are then

$$\begin{array}{ll} \text{old} & 100W + 100H \\ \text{new} & 50W + 125H \end{array}$$

so the new method is more profitable if and only if

$$50W + 125H < 100W + 100H$$

$$\Rightarrow 2W + 5H < 4W + 4H$$

$$\Rightarrow 2(W/H) + 5 < 4(W/H) + 4$$

$$\Rightarrow 1 < 2(W/H) \Rightarrow W/H > .50$$

But $W/H > .50$ says that the workers get back in wages more than half of the value of their output, i.e. the rate of surplus value is less than 100%. So it turns out that the plan parameter R plays an equivalent role to the rate of surplus value in the capitalist calculation.

This can be shown to hold more generally. Let d_0 and i_0 denote respectively the direct and indirect labor requirements for the old method, and d_1 and i_1 denote the direct and indirect requirements for the new. By the same reasoning as above we arrive at the following criteria for the superiority of the new method over the old, in the planned system using parameter R and the capitalist profitability calculation:

$$\begin{array}{ll} \text{planned:} & (d_0 - d_1)/(i_1 - i_0) > 1 + R \\ \text{capitalist:} & (d_0 - d_1)/(i_1 - i_0) > H/W \end{array}$$

That is, $(1+R)$ and H/W play an equivalent role, but

$$1 + R = H/W \Rightarrow R = H/W - 1 = (H - W)/W$$

and, given the definitions of H and W, $(H - W)/W$ corresponds to the rate of surplus value.

This raises a problem. In our example we use plausible numbers with a rate of surplus value in excess of 100 percent. Now if Samuelson's R is defined as above, i.e. $(1 + R) = \gamma\beta$ where γ is one plus the rate of growth of labor supply and β is one plus the rate of labor-saving technical progress, then R is clearly much less than 100% (probably more like 5%). Direct-labor saving methods which satisfy the social R criterion may well fail to satisfy the capitalist profitability criterion.

The sleight of hand in the Samuelson–Weizsäcker argument lies in presenting the use of a discount rate equal to the real rate of growth of the economy as if it were the standard bourgeois method of economic calculation. But of course the actual bourgeois profit rate is generally greatly in excess of the real growth rate. A substantial part—perhaps the greater part—of aggregate profit goes to meet the extravagant lifestyle of the upper classes and contributes nothing to economic growth.

Consider the effect of a uniform reduction of wages by 30% in an economy with an initial split of the working day 60/40 wages to surplus value: the rate of surplus value would rise from 66% to 150% and in the process a whole mass of labor intensive activities—sweated trades, fast food outlets, telephone cleaning services and assorted skivvying—would become economically viable. This would have taken place without any alteration in b (the rate of exogenous technical change) or in g (the rate of growth of the labor force). There is no socially rational basis for switching labor into these labor-intensive activities; the switch comes about solely due to the change in the distribution of income between classes in society. The example is not fanciful; one saw it work in reverse between 1939 and 1950 when a rise in the share of income going to workers meant that the middle classes could no longer afford private servants, and encouraged the market for domestic appliances. Hoovers and washing machines had been available, with little technical change, since the turn of the century but it was not worth buying them so long as maids could be hired for £2 a week.

This rise in wages did not mean that the rate of growth of the economy slowed down; on the contrary a higher cost of labor encourages the use of more machinery which in turn accelerates technical change. Historically, one could argue for an *negative* correlation between the rate of surplus value under capitalism and the rate of technical improvement. The classic example of this must be the USA in the 19th century, where the free availability of land held wages up and encouraged labor saving innovations, which in turn led to the US having the highest labor productivity in the world.

By contrast in a socialist economy, where the wasteful consumption of the rich has been done away with, the rate of surplus product and the rate of growth of the economy might be more closely related. In the case of an economy undergoing extensive growth—e.g. the USSR during initial industrialization—there will be a strong positive correlation between the two: when the rate of surplus product extraction is high, we might assume that R will also be. Under these circumstances it may be rational to use techniques that are more labor-intensive than a simple undated labor-time calculation would justify.

The large scale irrigation work done in China in the 1960s was largely accomplished using manual labor even though it might have been cheaper on a labor cost basis to use bulldozers. But the point was that the bulldozers did not exist whereas the labor power did. Even a simple planning in kind would reveal this. Consider what would happen if China had been using our proposed planning mechanism. The local communes would propose to build a dam and calculate the cheapest way of doing it in terms of labor. This could involve half a dozen JCBs and 10 workers. They submit this plan to the planning computers. These perform a physical balance operation and come out with the result that the activity is to be operated at a zero intensity level because there are not enough JCBs to go round all the communes that want them. The commune then puts in for a second attempt suggesting the use of 50 workers with picks and wheelbarrows. Since there are no material resource constraints, this is allowed to go through. What this shows is that the most labor saving alternative technique for a given task may not be feasible in an economy with severe shortages of machinery, so that local attempts to optimize the use of labor time may have to be overridden by global resource constraints.

In a developed industrial economy the situation is rather different. R is likely to be much lower than in an economy undergoing extensive industrialization as the size of the labor force does not change so fast. In this case errors due to using labor values for an initial calculation of what is the cheapest technique will be much smaller. They will certainly be far less than those induced by a 100% rate of surplus value in a capitalist economy.

Let us consider the effect of these errors: Goods with a long production period—wine in Samuelson's example—will sell for the same price as goods with a short period (grape juice). Use of our marketing algorithm (section 2.4 above) will result in somewhat more than the optimum quantity of wine being produced if the rate of growth of the labor force, g, is positive, or if the rate of labor-augmenting technical progress is > 0 and we use current costs rather than historic costs. If, on the other hand, g is relatively close to zero and we use historic labor costs, then the use of labor values will produce a resource allocation that is almost identical to the Samuelsonian optimum.

The issue is a serious one only when planners are dealing with projects with a very long time horizon—the Channel tunnel versus a new ferry, a tidal barrage versus a coal-fired power station. In such cases, the bourgeois method of calculation as used by the electricity authorities can lead to some very counter-intuitive results. For instance advocates of the Severn barrage in the UK say that using conventional accounting techniques the difference between assuming that the barrage will last 30 years or 60 years is only to reduce the cost per KWH of power from 6.05p to 6.03p. The second 30 years of by then almost free power have been depreciated out of existence by the discounted cash flow analysis. For a major engineering work like that one would anticipate a lifetime of more like one or two centuries; such longevity is made to appear irrelevant by the bourgeois method of accounting.

In conclusion, although the use of labor time in evaluating technical alternatives may lead to errors:

1. Serious errors will be caught by the system of material balances.

2. In an economy with a stable population the errors are small and can be removed by using historic labor costs.

3. These "errors" are with respect to a standard that itself produces some anomalous results in evaluating long-term proposals, and they are likely to be ecologically benign.

4. The errors are far less than those induced by the exploitation of cheap labor in a capitalist pricing system.

5 "Bourgeois prices" in the capitalist economy?

Almost all of the voluminous literature on the Marxian "transformation problem" is predicated on the assumption that, whether or not he succeeded, what Marx was trying to do in Part II of *Capital*, volume III—namely, derive a set of prices consistent with the equalization of the rate of profit across all capitals, i.e. "bourgeois prices" as in Samuelson—was correct. Those critics of Marx who argue that there is really no transformation problem (on the grounds that labor values are theoretically redundant—see Steedman, 1977) most emphatically share this assumption.

But there is a growing body of empirical evidence to indicate that price formation in capitalist economies can be modeled at least as well by simple labor values as it can by prices of production.[14] The theory of prices of production postulates that prices are set by the following equation (Steedman, 1977):

$$(1 + r)(p^m\mathbf{A} + ma) = p^m \qquad (3)$$

where $\mathbf{A}$ is the matrix of produced means of production, a the row vector showing the level of employment in each industry, r is a uniform rate of profit, p^m a row vector of money prices and m the money wage rate.

[14] See Farjoun and Machover, 1983; Shaikh, 1984; Petrovic, 1987; Ochoa, 1989; Valle Baeza, 1994; Cockshott, Cottrell and Michaelson, 1995; Cockshott and Cottrell, 1997. Cf. Freeman, 1998, for a skeptical assessment.

The crucial assumption here is the existence of a uniform profit rate r. This is clearly a rather forced assumption since in practice the profit rate is a random variable both within and between industries. In itself this not a particularly serious problem provided that the rate of profit is statistically independent of the capital–labor ratio, or organic composition (o) in Marxian terminology. It is this statistical independence of the rate of profit vis-à-vis the capital–labor ratio that distinguishes price of production theory from the simple labor theory of value. The latter predicts that industries with a high capital–labor ratio will have a lower rate of profit than those with an low capital–labor ratio, or in other words that r and o would be negatively correlated.

In a study based on the UK input–output table for 1984 (Cockshott and Cottrell, 1998) we found that the data were inconsistent with the theory of prices of production: there was a significant negative correlation between profit rates and organic compositions of capital as predicted by the simple labor theory of value. The results of that study were, however, open to the criticism that our data-set (based solely on input–output flows) lacked proper capital stock figures. Capital stock figures are somewhat easier to obtain for the USA, and we present below an analysis using these data.

5.1 United States data

Our data were drawn from the 1987 US input–output table along with BEA capital stock figures for the same year. The BEA give figures for plant and equipment at a higher level of aggregation than that employed in the input–output table. We therefore had to merge some rows and columns of the table to ensure that each industry had a distinct figure provided for the value of plant and equipment. The resulting table has 47 columns and 61 rows. The columns—which constitute a square submatrix along with the first 47 rows—represent the aggregated industry groups. The remaining rows consist of:

- Inputs for which there is no corresponding industry output listed such as “Educational and social services, and membership organizations” or “Noncomparable imports” (a total of 9 rows).
- “Compensation of employees”, which we treat as equivalent to variable capital.
- Items which form part of surplus value:
  - “Indirect business tax and nontax liability”;
  - “Other value added”—we treat this as being profit;
  - “Finance”—we treat this as corresponding to interest; and
  - “Real estate and royalties”, which we treat as corresponding to the category rent.

The BEA figures are for fixed capital; we assumed that in addition industries held stocks of work in progress valued at one month’s prime costs (excluding wages). The capital stock figures used were then taken as the sum of work in progress plus plant and machinery.

Modeling capital stocks is the logical dual of modeling turnover times. We are in effect assuming that for the aggregate capital the turnover time of circulating capital is one month. This assumption is based upon the heroic simplification that there exist 12 production periods per year corresponding to monthly salary payments, and that the total stocks of goods held in the production, wholesale and retail chain amount to one month’s sales. That is, we assume that the turnover time of variable capital is one month with wages paid in advance, and that circulating constant capital is purchased simultaneously with labor. (In the calculation of prices of production we assume wages are paid at the end of the month.) A more sophisticated study would look at company accounts for firms in each sector to build up a model of the actual stocks of work in progress. Industries operating just-in-time production will have considerably lower stocks and thus faster turnover. For other industries one month’s stocks may be an underestimate.

5.2 Correlations

We computed the total value of output, industry by industry, using the labor-value and price of production models. This gave two estimates for the aggregate price vector; the correlation matrix with observed prices is given in Table 3. Both estimates of the value of total industry output are highly correlated with market prices, but the labor-value estimates are marginally better.

Table 3: Correlation matrix of logs of estimates of total industry output for 47 sectors of US industry (P = observed price, E_1 = labor values, E_2 = prices of production)

	P	E_1	E_2
P	1		
E_1	0.971	1	
E_2	0.968	0.936	1

That prices of production are not clearly ahead of simple labor values in predicting market prices is comprehensible in terms of the observation that profit rates, counter to production price theory, are lower in industries with a high organic composition of capital. This is illustrated in Figure 3, which shows three sets of points:

1. the observed rate of profit, measured as s/C where C denotes capital stock;
2. the rate of profit that would be predicted on the basis of commodities exchanging at prices proportional to their labor values, i.e. $s'v/C$ where s' is the mean rate of exploitation in the economy as a whole; and
3. the rate of profit that would be predicted on the basis of prices of production (mean s/C).

Note that the observed rates of profit fall close to the rates that would be predicted by the simple labor theory of value (labeled the "vol. 1 rate" for short in Figure 3, since it corresponds to Marx's assumption in Volume I of *Capital* that prices are proportional to values). The exception is for a few industries with unusually high organic compositions > 10.

But what are these industries? They fall into two categories, each arguably exceptional. First there are the regulated utilities, electricity supply and gas supply. Electricity supply has an organic composition of 23.15, and displays a rate of profit half way between that predicted by the simple labor theory of value and that predicted by the price of production theory. The gas utilities have a rate of profit of 20% on an organic composition of 10.4; the labor theory of value would predict a profit rate of 7% and the production price theory 32%. In each case the industry is regulated, and of course the regulatory system builds in the assumption that the utilities should earn an average rate of profit. Second, there are industries of high organic composition in which rent plays a major role. At an organic composition of 16.4, the crude petroleum and natural gas industry has a rate of profit substantially in excess of that predicted by the labor theory of value, and approximating more closely that predicted by an equalization of the rate of profit. But this industry would be expected, on the basis of the Ricardian theory of differential rent, to sell its product above its mean value, and hence report above average profits. In a similar position we find the oil refining industry with an organic composition of 9.4. Oil production and oil refining have similar rates of profit, at 31% and 32%. Since the industry is vertically integrated, this would indicate that the oil monopolies chose to report their super profits as earned pro-rata on capital employed in primary and secondary production. In both cases, however, the super profit can be explained by differential rent.

The available data do not support the idea that prices of production serve as a more accurate predictor of actual market prices than labor values. Thus, to return to the theme of the previous section, if it were judged economically inefficient for a socialist economy to base its economic calculus on labor values rather than upon prices of production, the same inefficiency would appear to affect leading capitalist economies. In particular, the industries which, in the USA, conform most closely to the theoretical model of production price theory are those under government regulation. To that extent, production price theory may find its true application in state-regulated capitalism.

6 Conclusion

We have presented the outlines of a model of socialist planning which we claim would be efficient and responsive to popular needs. We have argued that such a system is technically feasible given the current state of computer technology, and we have defended the use of labor values in our proposed system from the charge that "bourgeois prices" (involving an equalized rate of profit) provide a superior means of economic calculation. From this perspective the failure of the Soviet model cannot be taken as synonymous with the failure of socialism: what failed in Russia was a particular form of planning, while other, superior forms of planning are possible.

A question may well suggest itself to the reader: Are we not being supremely arrogant in supposing that we have come up with an adequate scheme for central planning where the "best minds" in the USSR failed over a period of, say, 25 years? (That is, from 1960 or so, when the issue of reform of the planning system emerged, until the late 1980s when this whole conception was abandoned in favor of a transition to the market.) But it's not that we think ourselves smarter than the Soviet economists; rather we are not operating under the same constraints.[15] The two main intellectual inputs into our scheme are (a) a critical, non-dogmatic Marxism and (b) modern computer science. It was very difficult to combine these in the USSR, where "Marxism" so often served an obscurantist, anti-scientific function. Our views would probably have been considered deviationist by the guardians of orthodoxy… and at the same time naively socialist by those whose view of socialism was formed in the cynical Brezhnev years, and to whom Marxism was therefore nothing but a fossilized dogma.

[15] For an extended discussion of the differences between our proposals and the planning methods used in the former Soviet Union see Cottrell and Cockshott, 1993b.

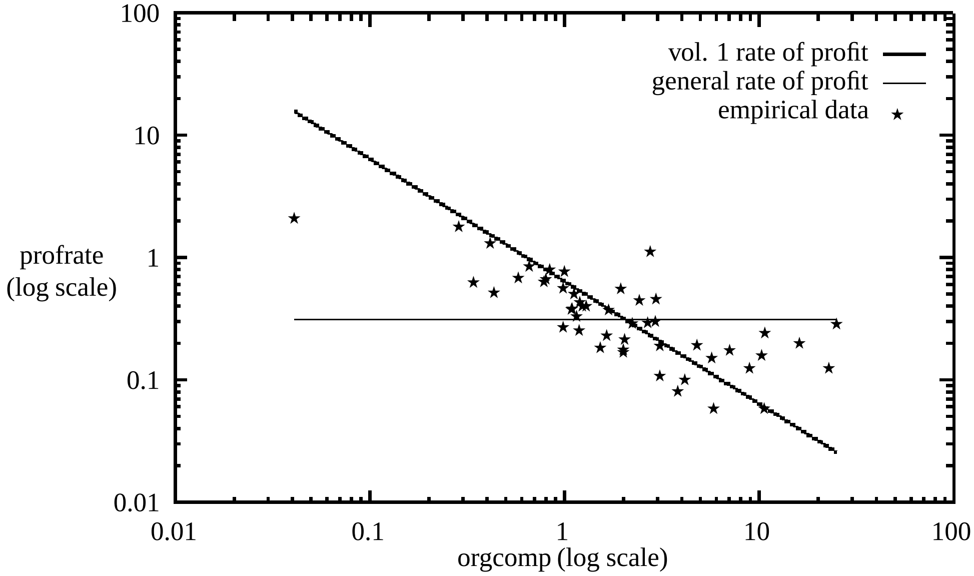

Figure 3: Relation between profit rates and organic composition

References

Cockshott, W. P. 1990. 'Application of artificial intelligence techniques to economic planning', *Future Computer Systems*, vol. 2, no. 4, pp. 429–43.

Cockshott, W. P. and Cottrell, A. 1993. *Towards a New Socialism*, Nottingham: Spokesman Books.

Cockshott, W. P. and Cottrell, A. 1997. 'Labour time versus alternative value bases: a research note', *Cambridge Journal of Economics*, vol. 21, no. 4, pp. 545–9.

Cockshott, W. P. and Cottrell, A. 1998. 'Does Marx Need to Transform?', in Bellofiore, R. (ed.), *Marxian Economics: A Reppraisal*, vol. 2, Basingstoke: Macmillan, pp. 70–85.

Cockshott, W. P., Cottrell, A. and Michaelson, G. 1995. 'Testing Marx: some new results from UK data', *Capital and Class*, 55, Spring, pp. 103–29.

Cottrell, A. and Cockshott, W. P. 1993a. 'Calculation, complexity and planning: the socialist calculation debate once again', *Review of Political Economy*, vol. 5, pp. 73–112.

Cottrell, A. and Cockshott, W. P. 1993b. 'Socialist planning after the collapse of the Soviet Union', *Revue Européene des Sciences Sociales*, vol. 31, pp. 167–185.

Dongarra, J, H. Meuer, H. and Strohmaier, E. 1997. 'TOP500 Report 1996', *Supercomputer*, no. 1.

Farjoun, E. and Machover, M. 1983. *Laws of Chaos*, London: Verso.

Freeman, A. 1998. 'The transformation of prices into values: comment on the chapters by Simon Mohun and Anwar M. Shaikh', in Bellofiore, R. (ed.), *Marxian Economics: A Reppraisal*, vol. 2, Basingstoke: Macmillan, pp. 270–275.

Freeman, A. and Carchedi, G. (eds) 1996. *Marx and Non-Equilibrium Economics*, Cheltenham: Edward Elgar.

Lange, O. 1938. 'On the economic theory of socialism', in Lippincott, B. (ed.), *On the Economic Theory of Socialism*, New York: McGraw-Hill.

Lange, O. 1967. 'The computer and the market', in Feinstein, C. (ed.), *Socialism, Capitalism and Economic Growth: Essays Presented to Maurice Dobb*, Cambridge: Cambridge University Press.

Marx, K. 1972. *Capital*, Volume 3, London: Lawrence and Wishart.

Marx, K. 1974. 'Critique of the Gotha Programme', in Fernbach, D. (ed.), *Karl Marx: The First International and After* (Political Writings, Volume 3), Harmondsworth: Penguin/New Left Review.

Marx, K. 1976. *Capital*, Volume 1, Harmondsworth: Penguin/New Left Review.

Mises, L. von 1935. 'Economic calculation in the socialist commonwealth', in Hayek, F. A., (ed.) *Collectivist Economic Planning*, London: Routledge & Kegan Paul.

Mises, L. von 1951. *Socialism*, New Haven: Yale University Press.

Nove, A. 1977. *The Soviet Economic System*, London: George Allen and Unwin.

Nove, A. 1983. *The Economics of Feasible Socialism*, London: George Allen and Unwin.

Ochoa, E. M. 1989. 'Values, prices, and wage–profit curves in the US economy', *Cambridge Journal of Economics*, vol. 13, no. 3, September, pp. 413–29.

Petrovic, P. 1987. 'The deviation of production prices from labour values: some methodology and empirical evidence', *Cambridge Journal of Economics*, vol. 11, no. 3, September, pp. 197–210.

Samuelson, P. and von Weiszäcker, C. 1972. 'A new labour theory of value for rational planning through the use of the bourgeois profit rate', in *The collected scientific papers of Paul A. Samuelson*, Vol. 3, Cambridge, Mass.: MIT Press.

Shaikh, A. 1984. 'The transformation from Marx to Sraffa', in A. Freeman and E. Mandel (eds) *Ricardo, Marx, Sraffa*, London: Verso, pp. 43–84.

Smolensky, P. 1986. 'Information Processing in dynamical systems: foundations of harmony theory', in Rumelhart, D. (ed.), *Parallel Distributed Processing*, vol. 1, Cambridge, MA: MIT Press.

Steedman, I. 1997. *Marx After Sraffa*, London: New Left Books.

Stiglitz, J. E. 1994. *Whither Socialism?* Cambridge, MA: The MIT Press.

Strumilin, S. G. 1977. 'K teorii tsenoobrazovaniya v usloviyakh sotsializma', in Akademiya Nauk USSR, editors, *Aktual'niye problemy ekonomicheskoy nauki v trudakh S. G. Strumilina*, Moscow: Nauka.

Valle Baeza, A. 1994. 'Correspondence between labor values and prices: a new approach', *Review of Radical Political Economics*, vol. 26, no. 2, pp. 57–66.

Varga, R. S. 1962. *Matrix Iterative Analysis*, Englewood Cliffs, NJ: Prentice Hall.

Yun, O. 1988. *Improvement of Soviet Economic Planning*, Moscow: Progress Publishers.