Value price transformation as a real process

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1 Introduction

Almost all of the voluminous literature on the Marxian 'transformation problem' is predicated on the assumption that, whether or not he succeeded, what Marx was *trying* to do in Part II of *Capital*, volume III—namely, derive a set of prices consistent with the equalization of the rate of profit across all capitals—was correct. Those neo-Ricardians who argue that there is really no 'transformation' problem as such (on the grounds that labour values are theoretically redundant—see Steedman, 1977) most emphatically share this assumption.

A rare exception to this orthodoxy is Farjoun and Machover's *Laws of Chaos* (1983). Farjoun and Machover, like Steedman, conclude that there is really no transformation problem, but for a very different reason, namely that the assumption of a tendency toward the equalization of the rate of profit is both empirically false and theoretically untenable. Rather, they claim, the predictions of the simple labour theory of value, as in Volume I of *Capital*, are in better accord with the facts.

Our aims in this paper are to explain this claim, to present some empirical data by means of which the claim may be assessed (based mainly on analysis of the UK input–output tables), and to offer some thoughts on the economic mechanisms that might be responsible for generating the observed data. We begin with a brief examination of the logic of the standard equalized-profit assumption.

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2 The tendency towards an equalized rate of profit

Supporters of the assumption of an equal rate of profit for the purposes of the theoretical analysis would surely admit that rates of profit, in any particular economy at any particular time, show quite a wide dispersion. Their claim is not that there actually exists a single rate of profit, but that there exists a definite *tendency* to produce equalization, and that for theoretical purposes it is legitimate to assume that this tendency is fully realized.

But what exactly is the status of such a tendency? On this theory, should we expect to see the dispersion of rates of profit narrowing over time in actual capitalist economies? If that is the idea, it seems to be empirically false. Farjoun and Machover produce evidence that the empirical frequency distribution of profit rates is broadly stable over time, with no observable tendency to collapse towards degeneracy. The alternative is to claim that the tendency towards equalization is something inherent in the process of competition among capitals, but that it is 'masked' by the continuous occurrence of external shocks or disturbances. This theory relies on a partitioning of the causes operating on the dispersion of profit rates. Internal to the logic of the system is a competitive process that drives towards equalization, while the dispersion-enhancing disturbing factors are exogenous. What are the latter factors? If they were sunspots, hurricances, earthquakes and so on, the theory would be coherent (but even so, if the net result of the endogenous equalization process and the exogenous shock process is the maintenance of a roughly steady degree of dispersion, the equalized-rate assumption would not be very useful for analysis of real economies). But surely the most *significant* factors making for increased dispersion of profit rates are just as endogenous to the process of capitalist competition, or rivalry, as the equalizing factors: the development and application of new nologies; the development of new products; the exploitation of new markets or new sources of supply of labour or raw materials.

In the classical analysis—shared by Smith, Ricardo and Marx—the primary force working towards equalization is the mobility of capital between sectors of the economy in response to profit-rate differentials. If industry X is showing above-average profit, capital will move in, increasing the supply of the product and hence driving down both price and profit-rate. If Industry Y shows below-average profit, capital will tend to exit the industry, reducing supply and hence raising price and profit-rate. This mechanism makes sense in itself, but (leaving aside the question of the conditions required for such migration to produce stable convergence on an equalized rate of profit) it represents only one aspect of capitalist competition, understood broadly as the restless search for the greatest possible profit. Admit the other aspects of inter-capitalist rivalry (alluded to above), and it becomes an empirical question whether competition produces (a) an actual tendency towards equalization, (b) a tendency towards ever greater

dispersion, or (c) a roughly stable probability distribution for the rate of profit. As we have noted, the available data favour conclusion (c).

Why, then, does the equalized-profit assumption exercise such a hold over theorists? It may be that there is a temptation to think of competition among productive capitals on the model of arbitrage in financial markets. But this model is very misleading. The equalization of returns on financial assets comes about almost instantaneously via revaluation of securities, while the equalization of returns on industrial capital is at best a very slow process, dependent upon on the rate of depreciation and the speed with which new production facilities can be financed, built, and brought into production. There is also the syndrome of looking for one's keys under the lamp post. Suppose the equalization assumption is false—all the same, how *else* is one supposed to derive determinate theoretical results? If one assumes a non-equalized set of profit rates, how can one reach any conclusions? The problem here, Farjoun and Machover suggest, is the restriction of the search to *determinate* results: a *stochastic* version of the labour theory of value can manage quite well without an equalized profit rate, and still generate interesting and testable predictions regarding the 'laws of motion' of capitalism.

3 The stochastic approach

Farjoun and Machover make a distinction between the realm of production, where matters are relatively determinate, and the realm of price-formation and profits, where the 'anarchy of the market' prevails and the relevant magnitudes must be thought of as random variables. The search for the 'correct' determinate linkages between these variables is displaced by an analysis of the relevant probability distributions, their respective degrees of dispersion and their interconnections. In this spirit we offer below a list of the most important distributions to be examined in order to assess the relative merits of the simple labour theory of value and the theory of prices of production (either Marxian or Sraffian).

1. The distribution of ratios of market prices to labour values, $f(\psi)$, where $\psi = P/\Lambda$. (P denotes market price and Λ denotes embodied labour-time.)
2. The distribution of rates of profit, $f(r)$, where $r = S/(C + V)$.[1]
3. The distribution of ratios of market prices to prices of production, $f(\phi)$, where $\phi = P/\Pi$. (Π denotes prices of production.)
4. The distribution of organic composition of capital, $f(o)$, where $o = C/(S+V)$.
5. The distribution of rates of surplus value, $f(s)$, where $s = S/(S + V)$.

[1] As usual, S, C and V denote, respectively, surplus value, constant capital and variable capital.

Table 1
Predictions of labour theory of value
and theory of prices of production

Distribution	LTV	TPP
$\psi = P/\Lambda$	narrow	wide
$r = S/(C+V)$	wide	narrow
$\phi = P/\Pi$	wide	narrow
$o = C/(S+V)$	wide	wide
$s = S/(S+V)$	narrow	wide

A word on the definitions of these distributions. Conceptually, $f(r)$ is the probability density function such that $\int_a^b f(r)\,dr$ gives the fraction of the total social capital earning a rate of profit $a < r < b$ percent. Similarly, $\int_a^b f(o)\,do$ gives the fraction of capital having an organic composition $a < o < b$, and $\int_a^b f(s)\,ds$ gives the fraction of capital displaying a rate of surplus value $a < s < b$. For the ratio of market price to value, $\int_a^b f(\psi)\,d\psi$ gives the fraction of the total social product (measured in embodied labour-time) exchanging for a price $a < \psi < b$ per unit of embodied labour; and $\int_a^b f(\phi)\,d\phi$ gives the fraction of the total product (measured in terms of its price of production) exchanging for an observed price $a < \phi < b$ per unit price of production.

Table 1 shows the respective predictions of the simple labour theory of value or LTV (understood as the claim that commodities exchange in proportion to the socially necessary labour time required to produce them) and the theory of prices of production or TPP (that is, the theory that prices are formed so as to ensure an equalized rate of profit), with regard to these distributions. In the table, 'narrow' indicates that the distribution in question ought, on the particular theory, to have a relatively small standard deviation (taken literally, the prediction in these cases is degeneracy of the distribution, but nobody expects to find that in practice). The entry 'wide' indicates that the theory places no restriction on the degree of dispersion of the distribution in question. In principle, the simple LTV restricts only the distribution of price-to-value ratios. Given the auxiliary assumption that the dispersion of wage rates across industries is relatively narrow, however, the LTV also predicts a narrow dispersion of rates of surplus value. We have in effect built this assumption into our measurements, by using the wage bill of each sector as a proxy for hours worked, hence the 'narrow' entry against s in the LTV column. The theory of prices of production restricts only the distributions of rates of profit and, correspondingly, the ratios of actual prices to prices of production.

4 The data

We now turn to the empirical probability distributions for these variables. Our data are derived from the UK input–output tables for 1984 (Central Statistical Office, 1988). S, C and V are all expressed in monetary terms. Values, Λ, and prices of production, Π, were calculated by an iterative procedure. The input–output tables give a single, discrete observation on S, C, V, P, Λ and Π for each sector of the economy. The statistics of interest (mean, standard deviation and coefficient of variation) were calculated from these discrete observations using appropriate weights. For instance, in calculating the standard deviation for $\psi = P/\Lambda$ the weight given to each sector is $w_i = \Lambda_i / \sum_i \Lambda_i$, while for the rate of profit the weight is $w_i = (C_i + V_i) / \sum_i (C_i + V_i)$. The graphs showing the shape of the various distributions were derived via the application of a convolution function to the discrete data. Let $\hat{x}_i$, $i = 1, \ldots, n$ denote the discrete observations on some variable of interest, x, for each of the n sectors in the input–output table. We compute the continuous pdf given by

$$f(x) = \sum_{i=1}^{n} w_i N_{\hat{x}_i, \sigma_c}(x)$$

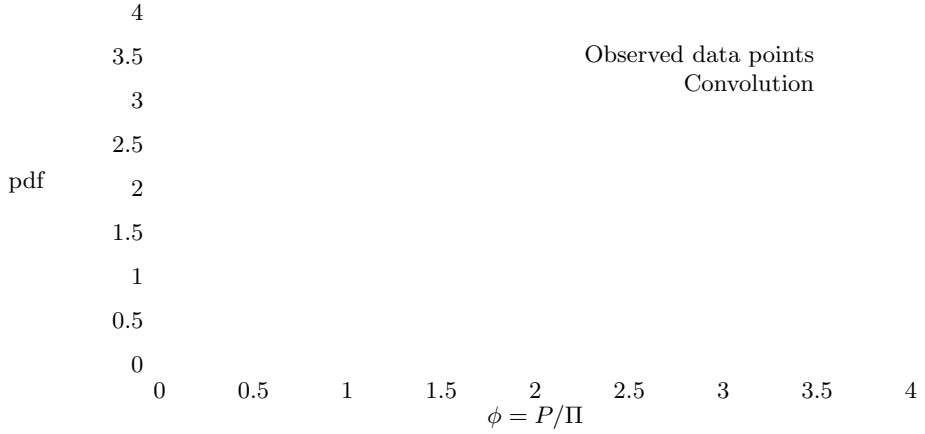
where $N_{\mu,\sigma_c}(x)$ is the value of the normal pdf, with mean μ and standard deviation σ_c, at point x. The assumption here is that each of the sectoral observations in fact represents the mean of a normal distribution. The degree of smoothing of the resulting curve depends on the value chosen for σ_c, the standard deviation employed in the convolving function.[2]

The UK tables comprise 101 sectors, four of which we excluded from our analysis (Agriculture, Extraction of oil and gas, Mineral oil processing, and Gas). The rationale for excluding these sectors is that they exhibit strong rent effects. Ricardo and Marx were very clear on how rent produces a deviation from the simple labour theory of value, and the rent effect is not at issue between the labour theory of value and the theory of prices of production. The rent effect is most apparent in the case of oil and gas extraction. Figure 1 shows, for reference, the distribution of ϕ, the ratio of market price to price of production, for all 101 sectors: the outlier to the right is the oil and gas sector. As regards the ratios of market prices to labour values, this induces a substantial second-round deviation for the oil processing and gas distribution sectors, since purchases from the oil and gas extraction sector account for approximately 50 percent and 30 percent, respectively, of the total input costs of these two sectors. In addition, the figure for income from employment in Agriculture is likely to understate substantially the labour used in that sector, due to the existence of family farms.

We should point out that due to the limitations of the available UK data, our figures for constant capital, C, are in flow rather than stock terms throughout.

[2] In the plots shown, σ_c was set at one-fifth of the standard deviation of the relevant distribution as a whole.

Figure 1: Ratios of actual prices to prices of production, 101 sectors



Correspondingly, our rates of profit, prices of production, and organic compositions, are all on a flow basis. We would have preferred to calculate these variables on a stock basis, and plan to do so, using data from the USA, in a future study.[3]

Table 2 shows the summary statistics for the observed distributions of o, r, s, ϕ and ψ, ranked in order of coefficient of variation. Thus o, our measure of organic composition, has the greatest degree of dispersion and ψ, the ratio of market prices to values, has the least. Note that the rate of profit, r, has a somewhat higher coefficient of variation than the rate of surplus value, s, and the ratios of actual prices to prices of production are slightly more broadly dispersed than the ratios of actual prices to values.

These findings are also illustrated by Figures 2, 3 and 4. Figure 2 shows both the sectoral data points and the convolved probability density function for the organic composition of capital. The outliers to the right are all sectors involved with food processing—Oils and fats, Sugar, Grain milling, and so on. These industries, it appears, take as their major input large quantities of agricultural commodities, and process them with relatively little labour input, per unit-value of raw materials. It may be noted, though, that even if these industries are left out of the calculation, the coefficient of variation for organic composition still exceeds that of any other variable under consideration. On the other hand, the great bulk of the pdf lies within the range 0.2 to 2.0, which represents a considerably narrower distribution than is implicit in many of the examples

[3]Two other details on our calculation methods: we evaluated the output of each sector at producer prices; and we counted the payment of interest to the banking sector as part of the surplus value in each sector.

Table 2
Summary statistics for
empirical distributions

Distribution	Mean	Std. Dev.	C. V.
$o = C/(S+V)$	0.846	0.636	0.752
$r = S/(C+V)$	0.211	0.129	0.608
$s = S/(S+V)$	0.315	0.134	0.423
$\phi = P/\Pi$	1.000	0.114	0.114
$\psi = P/\Lambda$	1.000	0.104	0.104

drawn up by Sraffian theorists.

Figure 2: Empirical distribution of organic composition

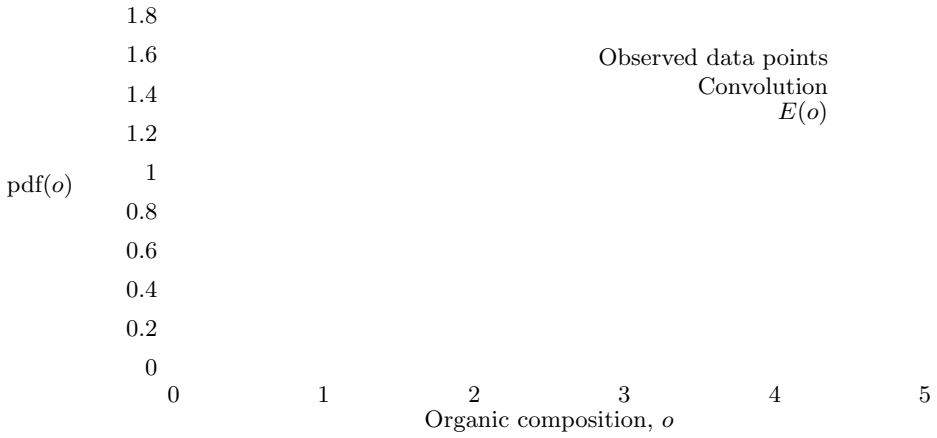


Figure 3 shows the convolved density functions for both the rate of profit and the rate of surplus value. As can be seen, the distribution of the rate of profit is far from degenerate. Figure 4 shows the distributions for the ratios of actual prices to values, and of actual prices to prices of production. It is easily seen that the degree of dispersion is quite similar in the two cases.[4]

[4]Both of these distributions have a mean of unity by construction. In effect, we have chosen a unit of measurement of labour so as to satisfy Marx's stipulation that the sum of prices equals the sum of values.

Figure 3: Distributions of rates of profit and of surplus value

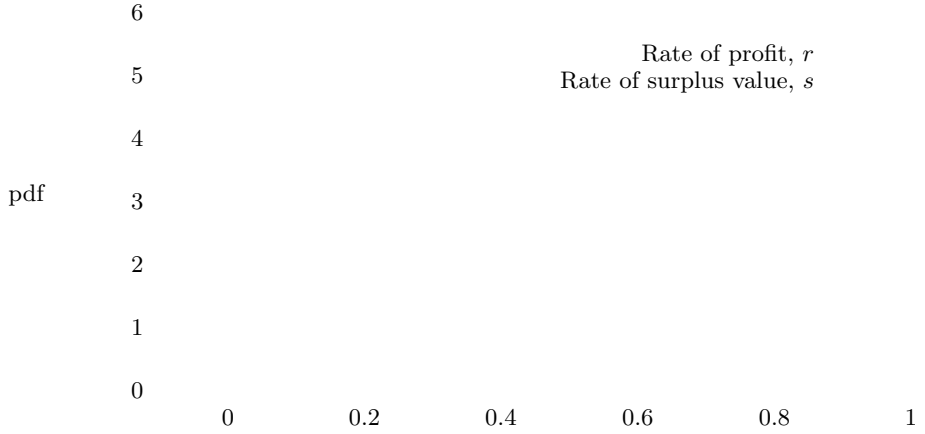
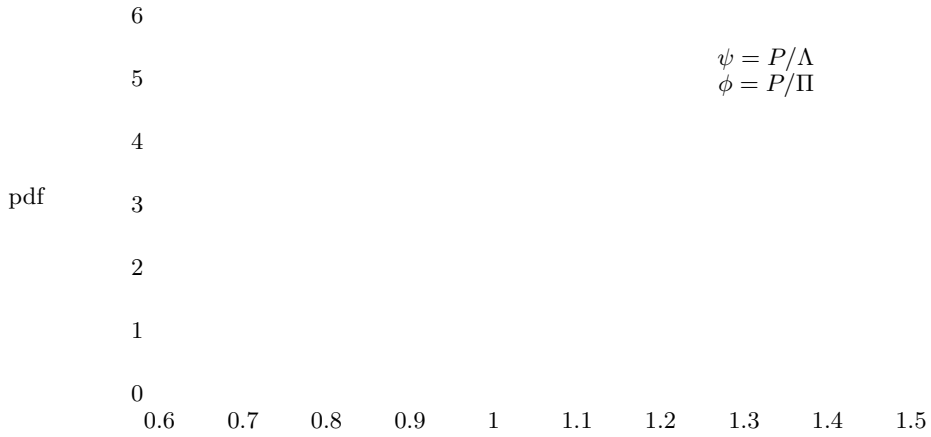


Figure 4: Ratios of actual prices to values, and to prices of production



5 Implications of the data

Let us first consider the implications of the above data for the theory of prices of production. As we noted, the rate of profit is far from actual equalization. On the other hand, the distribution of ratios of actual prices to prices of production is relatively tight. So can we say that the theory of prices of production holds as a reasonable approximation? Not really. There are some important anomalies in the data, from the point of view of this theory. Note that the ratio of market price to value can be be decomposed as follows:

$$\frac{P}{\Lambda} = \frac{P}{\Pi}\frac{\Pi}{\Lambda}.$$

In terms of logs, this can be re-written as

$$\log(P) - \log(\Lambda) = (\log(p) - log(\Pi)) + (\log(\Pi) - \log(\Lambda)),$$

which is to say that the deviation, in log terms, of price from value is the sum of (a) the deviation of price from price of production, and (b) the deviation of price of production from value. According to the theory of prices of production, these two elements ought to be independent of each other. Deviation (a) reflects the stochastic non-equalization of the rate of profit, while deviation (b) reflects the dispersion of the organic composition of capital; and it is the whole point of this theory that prices ought to be formed so as to eliminate any systemaic effect of differential organic composition on profit rates.

Now, if x and y are two indendently distributed random variables, and if $z = x + y$, then $\text{var}(z) = \text{var}(x) + \text{var}(y)$. The implication is that, provided the distribution of the organic composition is not degenerate—and it clearly is not, in the actual data—the standard deviation of P/Λ ought to be greater than that of P/Π. But this is not the case. It must be, then, that the distribution of profit rates is *not* in fact independent of the distribution of the organic composition of capital, which is to say that the theory of prices of production, even under a stochastic interpretation, is false.

A closely related anomaly from the standpoint of the price of production theory is the fact that the rate of surplus value (which is on this theory not subject to any equalization pressure) shows a *smaller* relative dispersion than the rate of profit.

A further perspective on these points is given by Table 3, which displays the correlation matrix for all of the variables under consideration. Note the negative correlation (statistically significant at the .005 level) between the rate of profit and the organic composition of capital. It is this negative correlation that explains how simple labour values are able to provide as good a fit to actual prices (actually, on our data, a slightly better fit) as prices of production.[5]

[5]The close fit between prices and labour values has been confirmed in a series of regression analyses, including Shaikh (1984), Petrovic (1987), Ochoa (1989), Valle Baeza (1994) and Cockshott, Cottrell and Michaelson (forthcoming, 1995).

Table 3
Correlation matrix

	o	r	s	ϕ	ψ
$o = C/(S+V)$	1.000				
$r = S/(C+V)$	−0.288	1.000			
$s = S/(S+V)$	0.369	0.517	1.000		
$\phi = P/\Pi$	−0.224	0.930	0.491	1.000	
$\psi = P/\Lambda$	0.423	0.569	0.579	0.663	1.000

Note: For a sample size of 96, the 1 per cent critical value of the correlation coefficent, $\hat{\rho}$, is 0.262.

Let us now turn to the simple labour theory of value. The first point to notice here is the support the theory receives from the observed narrow distributions of the price–value ratio and of the rate of surplus value. On the basis of very general statistical considerations, plus the assumption that there should be a very small probability (no more than $\frac{1}{1000}$) of a commodity selling for a price too low to cover the total wage-costs of producing it, Farjoun and Machover (1983, chapter 5) predict that the ratio of price to value should be distributed approximately normally, with a coefficient of variation of no more than $\frac{1}{6}$. From our data, it appears they were conservative: the C.V. is closer to $\frac{1}{10}$.

On the other hand, from this point of view, and given that there is quite a wide range of actual profit rates, why should a predictor of prices based upon the assumption of a uniform profit rate give a dispersion of ϕ that is narrower than than that of the rate of profit?

This is comprehensible in terms of the fact that the rate of profit is considerably less than 1. Since profits make up only about 20% of prices a 50% variation in the rate of profit will produce a variation of prices of about 10%. Thus we would expect the coefficient of variation of ϕ to be about $\frac{1}{5}$ that of r. This is in fact what we observe from Table 2.

In addition, the data seem to indicate that some *partial* equalization of the rate of profit is going on. Note that by this we do not mean simply that the equalization of the rate of profit is subject to random disturbance; rather, we mean that reality seems to fall roughly half way between the simple labour theory of value and the theory of prices of production—half way, that is, between Volumes I and III of *Capital*! Consider in this light some of the other entries in Table 3. There is a negative correlation between organic composition and profit rates: this is what would be predicted on the basis of the simple labour theory of value. But there is also a significant *positive* correlation between organic composition and the rate of surplus value (expressed in terms of money): this is predicted by the theory of prices of production. Thus, while there seems to be some tendency for capitals with higher than average organic composition

to realize a higher rate of surplus value, this effect is not strong enough to 'compensate' fully for their higher proportion of constant capital. Essentially the same story emerges from the positive correlation between organic composition and the price–value ratio, ψ. The fact that there *exists* a positive correlation is consistent with the price of production theory; but, again, the correlation is not *strong* enough to validate the theory. It is not strong enough to eliminate the negative correlation (statistically significant at the .05 level) between organic composition and the ratio of price to price of production, ϕ.

It appears that market prices behave under the influence of two competing attractors: values, and production prices. How can we explain this?

There is clearly a mathematical constraint operating between the dispersion of rates of surplus value and the dispersion of prices from values. They are either both wide or both narrow. Conceptually there are therfore three types of causal mechanisms that could be at play:

1. Suppose that for some as yet untheorised reason the simple labour theory of value — where labour input is measured in hours rather than indirectly as wages paid — holds. The narrow dispersion of ψ could then be an effect of the equalisation of wage rates between industries.

2. Alternatively there may be a mechanism that operates on the rate of surplus value directly, acting to limit its dispersion. One can conceive of three subprocesses that might work this way:

   (a) High wage rates in an industry would provide an incentive for manufacturers to improve productivity and thus restore the share of value going to capital. This would limit the degree to which workers could reduce the rate of surplus value through trades union struggle.

   (b) On the other hand, a high profit share in an industry strengthens the bargaining position of workers. Workers are both more willing to strike if they know their employers are exploiting them intensively, and it costs the employers more to resist a strike. This would limit the degree to which employers could increase the rate of surplus value.

   (c) If productivity based wage bargaining was common this would tend to stabilise the wage share.

   These would all limit the dispersion of s.

3. Finally it is possible that mechanisms (1) and (2) both occur at the same time.

Further empirical work would have to be done to determine which of these hypotheses is correct.

6 Conclusion

Recent investigations into that actual statistical properties of of prices in real capitalist economies, indicate that the century long debate on the appropriate way to transform values into production prices has been largely a wasted effort. In practice, production prices are no better predictors of market prices than are values, and the underlying hypothesis of an equalised rate of profit, on which the whole debate was based, is counter factual.

The debate was not a total ideological closure, in that formulations in terms of linear algebra are empirically testable. But one has to ask why it took so long before empirical investigations were done?

The necessary input-output data have been available for 40 or more years, and the computer nology to process them for 30. But empirical tests of the theories had to wait until the last decade. The practice of political economy has in this area fallen far short of scientific standards. It cannot be too strongly emphasised that theorisation in the absence of empirical data leads only to arid speculation, which, in a domain like political economy, will be driven primarily by ideological pressures.

References

Central Statistical Office, 1988, *Input-Output Tables for the United Kingdom 1984*, London: HMSO.

Cockshott, W. P., Cottrell, A. and Michaelson, G., 1995, 'Testing Marx: some new results from UK data', forthcoming in *Capital and Class*.

Farjoun, E. and Machover, M., 1983, *Laws of Chaos*, London: Verso.

Marx, K., 1971, *Capital*, Volume III, Moscow: Progress Publishers.

Ochoa, E. M., 1989, 'Values, prices, and wage–profit curves in the US economy', *Cambridge Journal of Economics*, vol. 13, no. 3, September, pp. 413–29.

Petrovic, P., 1987, 'The deviation of production prices from labour values: some methodology and empirical evidence', *Cambridge Journal of Economics*, vol. 11, no. 3, September, pp. 197–210.

Shaikh, A., 1984, 'The transformation from Marx to Sraffa', in A. Freeman and E. Mandel (eds) *Ricardo, Marx, Sraffa*, London: Verso, pp. 43–84.

Steedman, I., 1977, *Marx After Sraffa*, London: New Left Books.

Valle Baeza, A., 1994, 'Correspondence between labor values and prices: a new approach', *Review of Radical Political Economics*, vol. 26, no. 2, pp. 57–66.