

# Appendix

## TABLES

**Table I Trigonometric Functions**

Degrees	Radians	Sin	Tan	Cot	Cos		
0	.000	.000	.000		1.000	1.571	90
1	.017	.017	.017	57.29	1.000	1.553	89
2	.035	.035	.035	28.64	.999	1.536	88
3	.052	.052	.052	19.081	.999	1.518	87
4	.070	.070	.070	14.301	.998	1.501	86
5	.087	.087	.087	11.430	.996	1.484	85
6	.105	.105	.105	9.514	.995	1.466	84
7	.122	.122	.123	8.144	.993	1.449	83
8	.140	.139	.141	7.115	.990	1.431	82
9	.157	.156	.158	6.314	.988	1.414	81
10	.175	.174	.176	5.671	.985	1.396	80
11	.192	.191	.194	5.145	.982	1.379	79
12	.209	.208	.213	4.705	.978	1.361	78
13	.227	.225	.231	4.331	.974	1.344	77
14	.244	.242	.249	4.011	.970	1.326	76
15	.262	.259	.268	3.732	.966	1.309	75
16	.279	.276	.287	3.487	.961	1.292	74
17	.297	.292	.306	3.271	.956	1.274	73
18	.314	.309	.325	3.078	.951	1.257	72
19	.332	.326	.344	2.904	.946	1.239	71
20	.349	.342	.364	2.747	.940	1.222	70
21	.367	.358	.384	2.605	.934	1.204	69
22	.384	.375	.404	2.475	.927	1.187	68
23	.401	.391	.424	2.356	.921	1.169	67
24	.419	.407	.445	2.246	.914	1.152	66
		Cos	Cot	Tan	Sin	Radians	Degrees

(Table I is continued on the next page.)

**Table I Trigonometric Functions (continued)**

Degrees	Radians	Sin	Tan	Cot	Cos		
25	.436	.423	.466	2.144	.906	1.134	65
26	.454	.438	.488	2.050	.899	1.117	64
27	.471	.454	.510	1.963	.891	1.100	63
28	.489	.469	.532	1.881	.883	1.082	62
29	.506	.485	.554	1.804	.875	1.065	61
30	.524	.500	.577	1.732	.866	1.047	60
31	.541	.515	.601	1.664	.857	1.030	59
32	.559	.530	.625	1.600	.848	1.012	58
33	.576	.545	.649	1.540	.839	.995	57
34	.593	.559	.675	1.483	.829	.977	56
35	.611	.574	.700	1.428	.819	.960	55
36	.628	.588	.727	1.376	.809	.942	54
37	.646	.602	.754	1.327	.799	.925	53
38	.663	.616	.781	1.280	.788	.908	52
39	.681	.629	.810	1.235	.777	.890	51
40	.698	.643	.839	1.192	.766	.873	50
41	.716	.656	.869	1.150	.755	.855	49
42	.733	.669	.900	1.111	.743	.838	48
43	.750	.682	.933	1.072	.731	.820	47
44	.768	.695	.966	1.036	.719	.803	46
45	.785	.707	1.000	1.000	.707	.785	45
		Cos	Cot	Tan	Sin	Radians	Degrees

**Table II Greek Alphabet**

A $\alpha$	Alpha	N $\nu$	Nu
B $\beta$	Beta	$\Xi \xi$	Xi, Si
$\Gamma \gamma$	Gamma	O $\omicron$	Omicron
$\Delta \delta$	Delta	$\Pi \pi$	Pi
E $\epsilon$	Epsilon	P $\rho$	Rho
Z $\zeta$	Zeta	$\Sigma \sigma$	Sigma
H $\eta$	Eta	T $\tau$	Tau
$\Theta \theta$	Theta	$\Upsilon \upsilon$	Upsilon
I $\iota$	Iota	$\Phi \phi$	Phi
K $\kappa$	Kappa	X $\chi$	Chi
$\Lambda \lambda$	Lambda	$\Psi \psi$	Psi
M $\mu$	Mu	$\Omega \omega$	Omega

**Table III Exponential Functions**

$x$	$e^x$	$e^{-x}$	$x$	$e^x$	$e^{-x}$
0.0	1.00	1.00	3.0	20.1	.050
0.1	1.11	.905	3.1	22.2	.045
0.2	1.22	.819	3.2	24.5	.041
0.3	1.35	.741	3.3	27.1	.037
0.4	1.49	.670	3.4	30.0	.033
0.5	1.65	.607	3.5	33.1	.030
0.6	1.82	.549	3.6	36.6	.027
0.7	2.01	.497	3.7	40.4	.025
0.8	2.23	.449	3.8	44.7	.022
0.9	2.46	.407	3.9	49.4	.020
1.0	2.72	.368	4.0	54.6	.018
1.1	3.00	.333	4.1	60.3	.017
1.2	3.32	.301	4.2	66.7	.015
1.3	3.67	.273	4.3	73.7	.014
1.4	4.06	.247	4.4	81.5	.012
1.5	4.48	.223	4.5	90.0	.011
1.6	4.95	.202	4.6	99.5	.010
1.7	5.47	.183	4.7	110	.0091
1.8	6.05	.165	4.8	122	.0082
1.9	6.69	.150	4.9	134	.0074
2.0	7.39	.135	5.0	148	.0067
2.1	8.17	.122	5.1	164	.0061
2.2	9.02	.111	5.2	181	.0055
2.3	9.97	.100	5.3	200	.0050
2.4	11.0	.091	5.4	221	.0045
2.5	12.2	.082	5.5	245	.0041
2.6	13.5	.074	5.6	270	.0037
2.7	14.9	.067	5.7	299	.0033
2.8	16.4	.061	5.8	330	.0030
2.9	18.2	.055	5.9	365	.0027
			6.0	403	.0025

**Table IV Natural Logarithms**

$n$	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0*		7.697	8.391	8.796	9.084	9.307	9.489	9.643	9.777	9.895
1	0.000	0.095	0.182	0.262	0.336	0.405	0.470	0.531	0.588	0.642
2	0.693	0.742	0.788	0.833	0.875	0.916	0.956	0.993	1.030	1.065
3	1.099	1.131	1.163	1.194	1.224	1.253	1.281	1.308	1.335	1.361
4	1.386	1.411	1.435	1.459	1.482	1.504	1.526	1.548	1.569	1.589
5	1.609	1.629	1.649	1.668	1.686	1.705	1.723	1.740	1.758	1.775
6	1.792	1.808	1.825	1.841	1.856	1.872	1.887	1.902	1.917	1.932
7	1.946	1.960	1.974	1.988	2.001	2.015	2.028	2.041	2.054	2.067
8	2.079	2.092	2.104	2.116	2.128	2.140	2.152	2.163	2.175	2.186
9	2.197	2.208	2.219	2.230	2.241	2.251	2.262	2.272	2.282	2.293
10	2.303	2.313	2.322	2.332	2.342	2.351	2.361	2.370	2.380	2.389

\* Subtract 10 if  $n < 1$ ; for example,  $\ln 0.3 \approx 8.796 - 10 = -1.204$ .

Table V Powers and Roots

$n$	$n^2$	$\sqrt{n}$	$n^3$	$\sqrt[3]{n}$	$n$	$n^2$	$\sqrt{n}$	$n^3$	$\sqrt[3]{n}$
1	1	1.000	1	1.000	51	2,601	7.141	132,651	3.708
2	4	1.414	8	1.260	52	2,704	7.211	140,608	3.733
3	9	1.732	27	1.442	53	2,809	7.280	148,877	3.756
4	16	2.000	64	1.587	54	2,916	7.348	157,464	3.780
5	25	2.236	125	1.710	55	3,025	7.416	166,375	3.803
6	36	2.449	216	1.817	56	3,136	7.483	175,616	3.826
7	49	2.646	343	1.913	57	3,249	7.550	185,193	3.849
8	64	2.828	512	2.000	58	3,364	7.616	195,112	3.871
9	81	3.000	729	2.080	59	3,481	7.681	205,379	3.893
10	100	3.162	1,000	2.154	60	3,600	7.746	216,000	3.915
11	121	3.317	1,331	2.224	61	3,721	7.810	226,981	3.936
12	144	3.464	1,728	2.289	62	3,844	7.874	238,328	3.958
13	169	3.606	2,197	2.351	63	3,969	7.937	250,047	3.979
14	196	3.742	2,744	2.410	64	4,096	8.000	262,144	4.000
15	225	3.873	3,375	2.466	65	4,225	8.062	274,625	4.021
16	256	4.000	4,096	2.520	66	4,356	8.124	287,496	4.041
17	289	4.123	4,913	2.571	67	4,489	8.185	300,763	4.062
18	324	4.243	5,832	2.621	68	4,624	8.246	314,432	4.082
19	361	4.359	6,859	2.668	69	4,761	8.307	328,509	4.102
20	400	4.472	8,000	2.714	70	4,900	8.367	343,000	4.121
21	441	4.583	9,261	2.759	71	5,041	8.426	357,911	4.141
22	484	4.690	10,648	2.802	72	5,184	8.485	373,248	4.160
23	529	4.796	12,167	2.844	73	5,329	8.544	389,017	4.179
24	576	4.899	13,824	2.884	74	5,476	8.602	405,224	4.198
25	625	5.000	15,625	2.924	75	5,625	8.660	421,875	4.217
26	676	5.099	17,576	2.962	76	5,776	8.718	438,976	4.236
27	729	5.196	19,683	3.000	77	5,929	8.775	456,533	4.254
28	784	5.292	21,952	3.037	78	6,084	8.832	474,552	4.273
29	841	5.385	24,389	3.072	79	6,241	8.888	493,039	4.291
30	900	5.477	27,000	3.107	80	6,400	8.944	512,000	4.309
31	961	5.568	29,791	3.141	81	6,561	9.000	531,441	4.327
32	1,024	5.657	32,768	3.175	82	6,724	9.055	551,368	4.344
33	1,089	5.745	35,937	3.208	83	6,889	9.110	571,787	4.362
34	1,156	5.831	39,304	3.240	84	7,056	9.165	592,704	4.380
35	1,225	5.916	42,875	3.271	85	7,225	9.220	614,125	4.397
36	1,296	6.000	46,656	3.302	86	7,396	9.274	636,056	4.414
37	1,369	6.083	50,653	3.332	87	7,569	9.327	658,503	4.431
38	1,444	6.164	54,872	3.362	88	7,744	9.381	681,472	4.448
39	1,521	6.245	59,319	3.391	89	7,921	9.434	704,969	4.465
40	1,600	6.325	64,000	3.420	90	8,100	9.487	729,000	4.481
41	1,681	6.403	68,921	3.448	91	8,281	9.539	753,571	4.498
42	1,764	6.481	74,088	3.476	92	8,464	9.592	778,688	4.514
43	1,849	6.557	79,507	3.503	93	8,649	9.644	804,357	4.531
44	1,936	6.633	85,184	3.530	94	8,836	9.695	830,584	4.547
45	2,025	6.708	91,125	3.557	95	9,025	9.747	857,375	4.563
46	2,116	6.782	97,336	3.583	96	9,216	9.798	884,736	4.579
47	2,209	6.856	103,823	3.609	97	9,409	9.849	912,673	4.595
48	2,304	6.928	110,592	3.634	98	9,604	9.899	941,192	4.610
49	2,401	7.000	117,649	3.659	99	9,801	9.950	970,299	4.626
50	2,500	7.071	125,000	3.684	100	10,000	10.000	1,000,000	4.642

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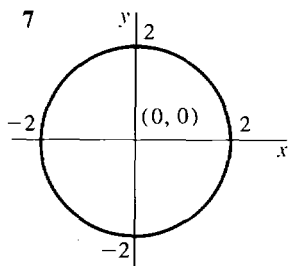
# ANSWERS TO SELECTED PROBLEMS

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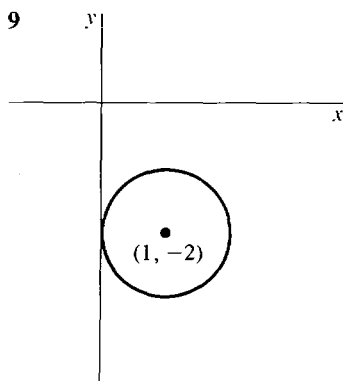
## Section 1.1

1 5 3  $\sqrt{13}$  5 13

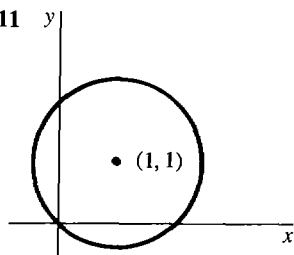
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9



11



13  $(x - 3)^2 + y^2 = 4$

15  $(x - 1)^2 + (y - \sqrt{3})^2 = 4, (x - 1)^2 + (y + \sqrt{3})^2 = 4$

17  $(x - 2)^2 + (y - 4)^2 = 25$

## Section 1.2

	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
1	-1/3	-1/6	0	1/6	1/3
3	-6	3/8	2	9/8	0
5	1	$1/\sqrt{2}$	0	*	*
7	2	1	1	1	2

9 yes

$$11 \quad f(2) = 7, f(t) = 1 + t + t^2, f(t + \Delta t) = 1 + (t + \Delta t) + (t + \Delta t)^2, \\ f(1 + t + t^2) = 1 + (1 + t + t^2) + (1 + t + t^2)^2, f(g(t)) = 1 + g(t) + (g(t))^2$$

$$13 \quad f(t) = t\sqrt{t}, f(t + \Delta t) = (t + \Delta t)^{3/2}, f(t^2) = |t^3|, f(\sqrt{t}) = t^{3/4}, f(g(t)) = (g(t))^{3/2}$$

15  $4\Delta x$ 

$$17 \quad \frac{1}{x^2 + 2x\Delta x + \Delta x^2} - \frac{1}{x^2} = -\frac{(2x + \Delta x)\Delta x}{x^2(x^2 + 2x\Delta x + \Delta x^2)}$$

$$19 \quad \sqrt{x + \Delta x} - \sqrt{x} = \frac{\Delta x}{\sqrt{x + \Delta x} + \sqrt{x}}$$

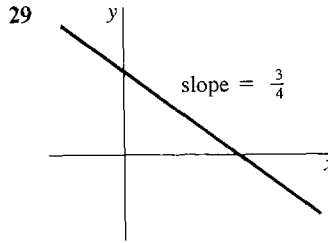
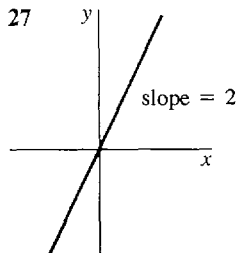
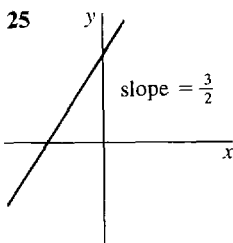
21 everywhere except  $x = 1$  and  $x = -1$     23  $x \geq 0$     25  $(-1, 1)$ 

## Section 1.3

$$1 \quad m = 1, y = x + 1 \quad 3 \quad \text{vertical}, x = -4 \quad 5 \quad m = -\frac{1}{3}, y = (-\frac{1}{3})x + 1$$

$$7 \quad m = 0, y = 3 \quad 9 \quad y = 2x - 3 \quad 11 \quad y = -(\frac{1}{2})x - \frac{7}{2} \quad 13 \quad y = 5x \quad 15 \quad y = 4$$

$$17 \quad v = 2, y = 2t \quad 19 \quad v = -\frac{1}{2}, y = -(\frac{1}{2})t + \frac{9}{2} \quad 21 \quad v = 0, y = 4 \quad 23 \quad y = 3t + 2$$

33  $(-c/a, 0), (0, -c/b)$ 

## Section 1.4 (no problem set)

## Section 1.5

- 1 infinitesimal    3 infinite    5 infinite    7 infinite    9 infinitesimal  
 11 infinitesimal    13 infinite    15 infinite    17 finite    19 finite    21 infinite  
 23 infinitesimal    25 infinite    27 finite    29 finite    31 finite    33 infinite  
 35 infinitesimal    37 infinite    39 finite

$$41 \quad (a) \varepsilon > \varepsilon^2 \quad (b) 1/\varepsilon^3 < 1/\varepsilon^4 \quad (c) H < H^2 \quad (d) \varepsilon < \sqrt{\varepsilon} \quad (e) H > \sqrt{H} \quad (f) \sqrt{H} > \sqrt[3]{H}$$

$$43 \quad (a) a = 0 \text{ and } b \neq 0 \quad (b) a \neq 0 \text{ and } b \neq 0, \text{ or } a = b = 0 \quad (c) a \neq 0 \text{ and } b = 0$$

## Section 1.6

$$1 \quad 2 \quad 3 \quad 2/5 \quad 5 \quad x^{12} \quad 7 \quad 0 \quad 9 \quad 2 \quad 11 \quad 3 \quad 13 \quad 0 \quad 15 \quad 1/2 \quad 17 \quad 1/2$$

$$19 \quad x + y \quad 21 \quad 3x^2 \quad 23 \quad 10 \quad 25 \quad -1/6 \quad 27 \quad 1 \quad 29 \quad 1/2 \quad 31 \quad -1/16$$

$$33 \quad \frac{1}{1 + \sqrt{2}} \text{ or } \sqrt{2} - 1$$

## Extra Problems for Chapter 1.

- 1  $\sqrt{122}$     3  $-5/3$     5  $y = -2x + 13$     7  $(x - 1)^2 + (y - 3)^2 = 5$   
 9 finite    11 infinite    13 infinite    15 infinitesimal    17 48  
 19  $4/7$     21  $-15/4$     23  $\frac{1}{\sqrt{x + \Delta x}} - \frac{1}{\sqrt{x}}$     ...

## Section 2.1

- 1  $2x$     3  $-4x$     5 4    7  $12t^2$     9  $5/(2\sqrt{u})$     11  $\frac{3}{2}\sqrt{x}$     13  $-2t^{-3}$   
 15  $-3y^{-2} + 4$     17  $a$     19  $a/(2\sqrt{ax + b})$     21  $2(3 - 2x)^{-2}$     23  $1/4$     25  $4t^3$   
 27 \$600, \$0,  $-\$1000$     29  $-3x^2$  if  $x \leq 0$ ,  $3x^2$  if  $x > 0$ .

## Section 2.2

- 1  $\Delta y = 2x \Delta x + \Delta x^2, dy = 2x \Delta x, \Delta y = dy + \Delta x^2$   
 3  $\Delta y = \frac{2 \Delta x}{\sqrt{x + \Delta x} + \sqrt{x}}, dy = \frac{\Delta x}{\sqrt{x}}, \Delta y = dy + \left[ \frac{2}{\sqrt{x + \Delta x} + \sqrt{x}} - \frac{1}{\sqrt{x}} \right] \Delta x$   
 5  $\Delta y = -\frac{\Delta x}{x(x + \Delta x)}, dy = -\frac{\Delta x}{x^2}, \Delta y = dy + \left[ \frac{1}{x^2} - \frac{1}{x(x + \Delta x)} \right] \Delta x$   
 7  $\Delta y = \left( 1 + \frac{1}{x(x + \Delta x)} \right) \Delta x, dy = \left( 1 + \frac{1}{x^2} \right) \Delta x, \Delta y = dy + \left[ \frac{1}{x(x + \Delta x)} - \frac{1}{x^2} \right] \Delta x$   
 9  $\Delta y = 4x \Delta x + 2 \Delta x^2, \Delta z = 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3, dy = 4x dx, dz = 3x^2 dx$     11  $2 dx$   
 13  $dx/(2\sqrt{x + 1})$     15  $a dx$     17  $(-2/x^2) dx$     19  $-\frac{1}{2}x^{-3/2} dx$   
 21  $d(y + z) = \left( \frac{1}{2\sqrt{x}} + 3 \right) dx, d\left(\frac{y}{z}\right) = -\frac{1}{6}x^{-3/2} dx$     23  $y = 4x - 4$     25  $y = 0$   
 27  $y = 3x - 4$     29  $y = -32x - 48$     31  $y = (2x_0)x - x_0^2$

## Section 2.3

- 1  $6x + 5$     3  $5(x + 8)^4$     5  $-3(4 - t)^2$     7  $6x(x^2 + 5)^2$     9  $-12x(6 - 2x^2)^2$   
 11  $24x^2(1 - 4x^3)^{-3}$     13  $5(1 + x^{-2})$     15  $-8(4x - 1)(2x^2 - x + 3)^{-3}$   
 17  $t^{-2}(1 + t^{-1})^{-2}$  or  $(t + 1)^{-2}$     19  $3(8t - 2)(4t^2 - 2t + 1)^{-2}$     21  $-3x^2 + 5x - 1$   
 23  $6(3t^2 + 1)(2t - 4)^2 + 6t(2t - 4)^3$  or  $6(2t - 4)^2(5t^2 - 4t + 1)$     25  $-2(x - 1)^{-2}$   
 27  $4x(x^2 + 1)^{-2}$     29  $(s^2 - 6s + 7)(s - 3)^{-2}$     31  $-5(3x - 4)^{-2}$     33 0  
 35  $6(x^2 + 1)(2x^2 - 1) + 12x(x^2 + 1)(2x + 3) + 6x(2x^2 - 1)(2x + 3)$   
 37  $-(4x - 5)(2x + 1)^{-2}(x - 3)^{-2}$     39  $2(2x + 1)^{-2}[(2x + 1)^{-1} + 3]^{-2}$   
 41  $4x(2x + 1)^3(x^2 + 1) + 6(2x + 1)^2(x^2 + 1)^2$  or  $2(2x + 1)^2(x^2 + 1)(7x^2 + 2x + 3)$   
 43  $\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$     45  $\frac{dy}{dx} = 4\frac{du}{dx} + 2v\frac{dv}{dx}$     47  $\frac{dy}{dx} = -\left[ u\frac{dv}{dx} + v\frac{du}{dx} \right] (uv)^{-2}$   
 49  $y = 6x - 2$     51  $b = -2, c = 4$

## Section 2.4

- 1  $(9y^2 + 2)^{-1}$     3  $-\frac{1}{4}y^{-1}$     5  $-\frac{1}{2}y^{-1}(y^2 + 2)^2$  or  $-1/(2x^2y)$     7  $\frac{4}{3}x^{1/3}$   
 9  $-\frac{1}{\sqrt{x}(\sqrt{x} - 1)^2}$     11  $\frac{2}{3}x^{-2/3} + \frac{8}{3}x^{-1/3} + 6$     13  $-2(\frac{2}{3}x^{2/3} - 1)(x^{5/3} - x)^{-3}$   
 15  $(y^{-2/3} + 2)^{-1}$     17  $y = \frac{1}{k}(x - c), \frac{dy}{dx} = \frac{1}{k}$     19  $y = \sqrt{\frac{x - 1}{2}}, \frac{dy}{dx} = \frac{1}{2\sqrt{2}\sqrt{x - 1}}$   
 21  $y = (x + 3)^{1/4}, \frac{dy}{dx} = \frac{1}{4}(x + 3)^{-3/4}$

$$23 \quad y = \left[ \frac{\sqrt{4x-3}-1}{2} \right]^{1/2}, \frac{dy}{dx} = (4y^3 + 2y)^{-1} \text{ or } \frac{dy}{dx} = \frac{1}{2} \left[ \frac{\sqrt{4x-3}-1}{2} \right]^{-1/2} (4x-3)^{-1/2}$$

$$25 \quad y = \left[ \frac{\sqrt{1+8x}-1}{4} \right]^2 = \frac{1+4x-\sqrt{1+8x}}{8}, \frac{dy}{dx} = \left( \frac{1}{2\sqrt{y}} + 2 \right)^{-1}$$

$$\text{or } \frac{dy}{dx} = \frac{1}{2} - \frac{1}{2\sqrt{1+8x}}$$

## Section 2.5

$$1 \quad -2 \sin \theta \cos \theta \quad 3 \quad 2 \cos x - 3 \sin x \quad 5 \quad \sin z(\cos z)^{-2} \quad 7 \quad n \sin^{n-1} \theta \cos \theta$$

$$9 \quad t \cos t + \sin t \quad 11 \quad e^x + xe^x \quad 13 \quad (2 \ln x)/x \quad 15 \quad e^x/x + e^x \ln x$$

$$17 \quad -e^v \sqrt{v} + (1 - e^v)/2\sqrt{v} \quad 19 \quad x^n/x + nx^{n-1} \ln x$$

$$21 \quad y - 1/2 = (\sqrt{3}/2)(x - \pi/6) \quad 23 \quad y = x - x/e$$

## Section 2.6

$$1 \quad \frac{1}{2}(x+2)^{-1/2} \quad 3 \quad -\frac{1}{2}(5-x)^{-1/2} \quad 5 \quad -\frac{3}{2}(2+3x)^{-3/2} \quad 7 \quad 2(6x+1)^{-2/3}$$

$$9 \quad x(x^2+1)^{-1/2} \quad 11 \quad 3 \cos(3x) \quad 13 \quad -2x^{-3} \cos(x^{-2}) \quad 15 \quad 4e^{4x}$$

$$17 \quad -\sin x e^{\cos x} \quad 19 \quad -e^x \sin(e^x) \quad 21 \quad -40(1-4x)^9$$

$$23 \quad -2x \cos(1-x^2) + 2 \cos(2x-1)$$

$$25 \quad \frac{e^{\sin x} \cos x}{2\sqrt{\sin x}} \quad 27 \quad -\frac{1}{3}x(1+\sqrt{x^2-1})^{-4/3}(x^2-1)^{-1/2}$$

$$29 \quad -12[1+(x^3+1)^{-1}]^3 x^2(x^3+1)^{-2}$$

$$31 \quad \frac{1}{3}[\sqrt{x^2-1} + \sqrt{x^2+1}]^{-2/3}[x(x^2-1)^{-1/2} + x(x^2+1)^{-1/2}]$$

$$33 \quad 6x \cos(2x-1) + 3 \sin(2x-1)$$

$$35 \quad -\frac{\cos(3t)}{\sin(3t)} \quad 37 \quad \frac{2 \cos(2t)}{\cos t}$$

$$39 \quad 2t(t+1) \quad 41 \quad \sqrt{t^2-4}/\sqrt{t^2+4}$$

$$43 \quad (t+1)^{2/3}(t+2)^{-2/3} \quad 45 \quad (9/2)t$$

## Section 2.7

$$1 \quad 2x^{-3} \quad 3 \quad -10(x+1)^{-3} \quad 5 \quad -(1/4)x^{-3/2} + (3/4)x^{-5/2} \quad 7 \quad (3/4)t^{-1/2}$$

$$9 \quad -\sin x \quad 11 \quad -AB^2 \sin(Bx) \quad 13 \quad a^2 e^{ax} \quad 15 \quad -x^{-2}$$

$$17 \quad -2(t^2+1)^{-2} + 8t^2(t^2+1)^{-3} \quad 19 \quad -14(x+2)^{-3}$$

$$21 \quad -\frac{1}{4}x(x+1)^{-3/2} + (x+1)^{-1/2} \text{ or } (\frac{3}{4}x+1)(x+1)^{-3/2}$$

$$23 \quad \frac{3}{4}t^{1/2}(t+3)^{-5/2} - \frac{1}{2}t^{-1/2}(t+3)^{-3/2} - \frac{1}{4}t^{-3/2}(t+3)^{-1/2} \text{ or } -(3t^{-1/2} + \frac{3}{4}t^{-3/2})(t+3)^{-5/2}$$

$$25 \quad -6t^{-4} \quad 27 \quad 3 \frac{d^2u}{dx^2} \quad 29 \quad 2u \frac{d^2u}{dx^2} + 2 \left( \frac{du}{dx} \right)^2$$

## Section 2.8

$$1 \quad -(y/x) \quad 3 \quad -(x/y)^2 \quad 5 \quad \frac{-y}{2y+x} \quad 7 \quad -(y/x)^3 \quad 9 \quad -\frac{2x+3y}{3x+2y} \quad 11 \quad \frac{5x^4}{2y-1}$$

$$13 \quad \frac{y}{2y-x} \quad 15 \quad \frac{6xy-y^2-1}{2xy-3x^2} \quad 17 \quad \frac{y \cos(xy)}{1-x \cos(xy)} \quad 19 \quad -\frac{1}{2 \sin y \cos y}$$

$$21 \quad \frac{y}{1-2y} \quad 23 \quad ye^x \quad 25 \quad 2(6y^2+4xy-3)^{-1}$$



- 27 at (1, 2),  $y' = -4/5$ ; at (-1, 3),  $y' = -1/5$   
 29 at (2, 1),  $y' = 2$ ; at (2, -1),  $y' = -2$ ; at  $(\sqrt{3}, 0)$ , vertical  
 31  $-2/3$     33  $e$     35  $\frac{2x}{y}, \frac{2y^2 - 4x^2}{y^3}$

### Extra Problems for Chapter 2

- 1  $12x^2 - 2$     3 8    5  $\Delta y = \frac{1}{(x + \Delta x)^3} - \frac{1}{x^3} = -\frac{3x^2 + 3x\Delta x + \Delta x^2}{x^3(x + \Delta x)^3} \Delta x$ ,  
 $dy = -3x^{-4} \Delta x$     7  $(2x - 2x^{-3}) dx$     9  $y = -x$     11  $-9x^2 - 5$   
 13  $9t^2 + 8t - 15$     15  $(-2v^4 + 10v^3 - 3v^2 - 16v + 20)(v^3 - 4)^{-2}$   
 17  $\frac{1}{2}x^{-1/2} + 6x^{1/2}$     19  $\frac{1}{3}x^{-2/3} + \frac{1}{4}x^{-5/4}$     21  $(2y + \frac{1}{2}y^{-1/2})^{-1}$  or  $\frac{2\sqrt{y}}{4y\sqrt{y+1}}$   
 23  $\frac{-3}{2\sqrt{1-3x}}$     25  $-\frac{5}{2}(5x + 4)^{-3/2}$     27  $-(3 + \frac{1}{2}t^{-1/2})(t^{-2} + 2t)^{-1}$   
 29  $-4(4x - 1)^{-3/2}$     31  $v = \frac{3t + 6}{2\sqrt{t + 3}}, a = \frac{3t + 12}{4(t + 3)^{3/2}}$     33  $-\frac{3y^3 + 6x^2y}{9xy^2 + 2x^3}$   
 35  $f'(x) = \begin{cases} 2x & \text{if } |x| > 1 \\ \text{undefined} & \text{if } |x| = 1 \\ -2x & \text{if } |x| < 1 \end{cases}$

### Section 3.1

- 1  $p = 4\sqrt{A}$     3  $V = S^{3/2}/(6\sqrt{\pi})$     5  $z = \sqrt{x^4 - x^2 + 1}$     7  $V = (s - 2x)^2x$   
 9  $s = 3y/(10 - y), 0 < y < 10$     11  $b/16$     13  $A = (1 + t)(w - 2t)$   
 15  $P(x) = x(100 - \sqrt{x}) - 10x$  or  $P(x) = x(90 - \sqrt{x})$

### Section 3.2

- 1  $100 \text{ cm}^2/\text{sec}$     3  $-2 \text{ in.}^2/\text{sec}$     5  $10\sqrt{61} \text{ mph}$     7  $10/3 \text{ in.}^2/\text{sec}$   
 9  $40(3g)^{1/2} \text{ ft/sec}$  where  $g = 32$ , or  $160\sqrt{6} \text{ ft/sec}$     11  $-120 \text{ mph}$   
 13  $\frac{800 \cdot 400}{\sqrt{(800)^2 + 6^2}} = \frac{160,000}{\sqrt{160,009}} \text{ mph}$     15  $-2 \text{ ft/sec}$   
 17  $1,000,000t - 300,000\sqrt{t} \text{ people/year}$     19  $\$60 \text{ per person per year}$   
 21  $(55/144)\pi \text{ in.}^2/\text{min}$     23  $-0.01$     25  $2 - (x/500)$     27  $-100 \text{ radians/hr}$   
 29 22

### Section 3.3

- 1 53    3 does not exist    5 does not exist    7  $-1/(4\sqrt{2})$     9  $1/3$   
 11 does not exist    13 does not exist    15 1    17  $(1 + \sqrt{2})^{1/2}$     19 1  
 21  $-1/2$     23  $2x$     25  $1/(2\sqrt{t})$     27  $3x^2$     29  $-2$     31 does not exist

### Section 3.4

- 1  $R$     3  $> -2$     5  $R$     7 all  $x$  except  $x = 0, x = -1$     9  $-2 < x < 2$   
 11 all  $x$  except  $x = (-1 \pm \sqrt{5})/2$  and  $x = -1$   
 13 all  $x$  except  $x = 2, x = 3$     15  $x < i$

## Section 3.5

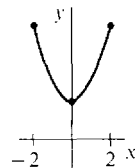
- 1 0, min    3 0, min    5 0, neither    7  $-1/3$ , min    9 0, min    11 0, max  
 13 0, neither    15  $-1/(2^{1/3})$ , min    17  $3/2$ , min    19 0, max    21  $\pi$ , neither  
 23 0, max    25 0, min    27  $-1$ , min    29 1, min    31 3, min    33 0, max  
 35  $1/2$ , max    37  $1/\sqrt{17}$  at  $(4/17, 1/17)$     39  $1 - m$  at  $x = 1$   
 41 max of  $3/4$  at  $x = 2$ , min of  $-1/4$  at  $x = -2$   
 43 max of  $3/(4 \cdot 3^{1/4})$  at  $x = 1/(3^{1/4})$ , min of  $-3/(4 \cdot 3^{1/4})$  at  $x = -1/(3^{1/4})$

## Section 3.6

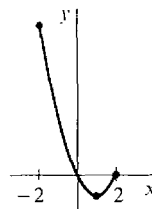
- 1  $x = 20/3, y = 40/3$     3  $x = 50, y = 1$     5  $x = 3/4, y = 1/4, x^3y = 27/256$   
 7 width =  $10\sqrt{3}$  in., length =  $40\sqrt{3}$  in.    9 base = height =  $1/\sqrt{2}$   
 11 side of square =  $10/3$  in., height of triangle =  $10/3$  in.    13 area =  $r^2$   
 15 base of radius  $2/3$ , height 1    17 base of radius  $4/(3\pi)$ , height  $4/3$     19 50  
 21  $x = 50,000, p = \$5$     23 256 sec    25  $1/2$     27 base 4, height 2  
 29  $1/\sqrt{17}$  at  $x = 4/17, y = 1/17$     31 height  $(3/2) \cdot$  side of base  
 33  $r = (2\pi)^{-1/3}, h = (4/\pi)^{1/3}$     35  $x = (a + b)/2$   
 37  $A = 2/r, dA/dr$  is never zero,  $0 < r < \infty$     39  $10\sqrt[3]{2}$  hours    41 62,500

## Section 3.7

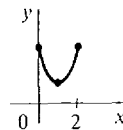
1	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	-2	6	-4	+	decr.	∪
	0	2	0	+	min.	∪
	2	6	4	+	incr.	∪



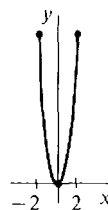
3	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	-2	8	-6	+	decr.	∪
	1	-1	0	+	min.	∪
	2	0	2	+	incr.	∪



5	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	0	3	-4	+	decr.	∪
	1	1	0	+	min.	∪
	2	3	4	+	incr.	∪

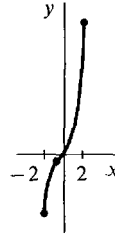


7	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	-2	16	-32	+	decr.	∪
	0	0	0	+	min.	∪
	2	16	32	+	incr.	∪



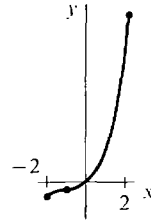
9

$x$	$f(x)$	$f'(x)$	$f''(x)$		
-2	-6	9	-	incr.	$\cap$
$-\frac{1}{3}$	$-\frac{7}{27}$	$\frac{2}{3}$	0	incr.	infl.
2	14	17	+	incr.	$\cup$



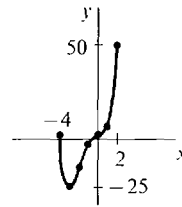
11

$x$	$f(x)$	$f'(x)$	$f''(x)$		
-2	$-\frac{2}{3}$	1	-	incr.	$\cap$
-1	$-\frac{1}{3}$	0	0	horiz.	infl.
2	$8\frac{2}{3}$	9	+	incr.	$\cup$



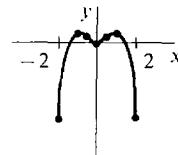
13

$x$	$f(x)$	$f'(x)$	$f''(x)$		
-4	2	-64	+	decr.	$\cup$
-3	-25	0	+	min.	$\cup$
-2	-14	16	0	incr.	infl.
-1	-1	8	-	incr.	$\cap$
0	2	0	0	horiz.	infl.
1	7	16	+	incr.	$\cup$
2	50	96	+	incr.	$\cup$



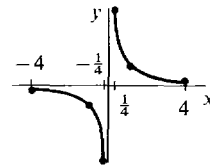
15

$x$	$f(x)$	$f'(x)$	$f''(x)$		
-2	-4	12	-	incr.	$\cap$
-1	$\frac{1}{3}$	0	-	max.	$\cap$
$-1/\sqrt{3}$	$\frac{5}{18}$	$-\frac{4}{3}\sqrt{3}$	0	decr.	infl.
0	0	0	+	min.	$\cup$
$1/\sqrt{3}$	$\frac{5}{18}$	$\frac{4}{3}\sqrt{3}$	0	incr.	infl.
1	$\frac{1}{3}$	0	-	max.	$\cap$
2	-4	-12	-	decr.	$\cap$



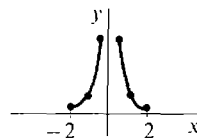
17

$x$	$f(x)$	$f'(x)$	$f''(x)$		
-4	$-\frac{1}{4}$	$-\frac{1}{16}$	-	decr.	$\cap$
-1	-1	-1	-	decr.	$\cap$
$-\frac{1}{4}$	-4	-16	-	decr.	$\cap$
$\frac{1}{4}$	4	-16	+	decr.	$\cup$
1	1	-1	+	decr.	$\cup$
4	$\frac{1}{4}$	$-\frac{1}{16}$	+	decr.	$\cup$



19

$x$	$f(x)$	$f'(x)$	$f''(x)$		
-2	$\frac{1}{4}$	$\frac{1}{4}$	+	incr.	$\cup$
-1	1	2	+	incr.	$\cup$
$-\frac{1}{2}$	4	16	+	incr.	$\cup$
$\frac{1}{2}$	4	-16	+	decr.	$\cup$
1	1	-2	+	decr.	$\cup$
2	$\frac{1}{4}$	$-\frac{1}{4}$	+	decr.	$\cup$



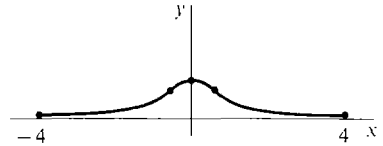
21

$x$	$f(x)$	$f'(x)$	$f''(x)$		
0	-1	2	-	incr.	$\cap$
1	0	$\frac{1}{2}$	-	incr.	$\cap$
10	$\frac{9}{11}$	$\frac{2}{121}$	-	incr.	$\cap$



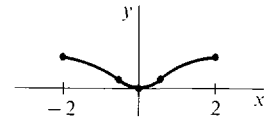
23

$x$	$f(x)$	$f'(x)$	$f''(x)$		
-4	$\frac{1}{17}$	.03	+	incr.	$\cup$
$-1/\sqrt{3}$	$\frac{3}{4}$	1.9	0	incr.	infl.
0	1	0	-	max.	$\cap$
$1/\sqrt{3}$	$\frac{3}{4}$	-1.9	0	decr.	infl.
4	$\frac{1}{17}$	-0.03	+	decr.	$\cup$



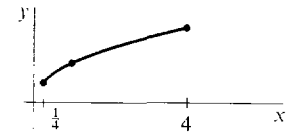
25

$x$	$f(x)$	$f'(x)$	$f''(x)$		
-2	$\frac{4}{5}$	$-\frac{2}{25}$	-	decr.	$\cup$
$-1/\sqrt{3}$	$\frac{1}{4}$	$-\sqrt{3}/2$	0	decr.	infl.
0	0	0	+	min.	$\cup$
$1/\sqrt{3}$	$\frac{1}{4}$	$\sqrt{3}/2$	0	incr.	infl.
2	$\frac{4}{5}$	$\frac{2}{25}$	-	incr.	$\cap$



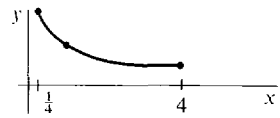
27

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\frac{1}{4}$	$\frac{1}{2}$	1	-	incr.	$\cap$
1	1	$\frac{1}{2}$	-	incr.	$\cap$
4	2	$\frac{1}{4}$	-	incr.	$\cap$



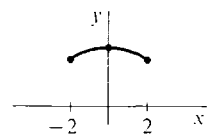
29

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\frac{1}{4}$	2	-4	+	decr.	$\cup$
1	1	$-\frac{1}{2}$	+	decr.	$\cup$
4	$\frac{1}{2}$	$-\frac{1}{16}$	+	decr.	$\cup$



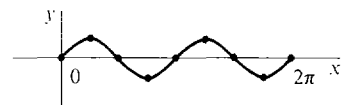
31

$x$	$f(x)$	$f'(x)$	$f''(x)$		
-2	$\sqrt{5}$	$2/\sqrt{5}$	-	incr.	$\cap$
0	3	0	-	max.	$\cap$
2	$\sqrt{5}$	$-2/\sqrt{5}$	-	decr.	$\cap$



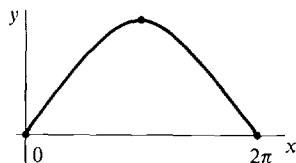
33

$x$	$f(x)$	$f'(x)$	$f''(x)$		
0	0	1	0	incr.	infl.
$\pi/4$	$\frac{1}{2}$	0	-	max.	$\cap$
$\pi/2$	0	-1	0	decr.	infl.
$3\pi/4$	$-\frac{1}{2}$	0	+	min.	$\cup$
$\pi$	0	1	0	incr.	infl.

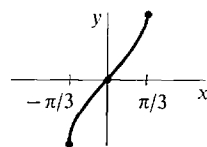


repeats

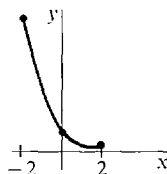
35	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	0	0	$\frac{3}{2}$	0	incr.	infl.
	$\pi$	3	0	-	max.	$\cap$
	$2\pi$	0	$-\frac{3}{2}$	0	decr.	infl.



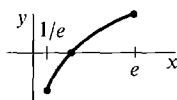
37	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$-\pi/3$	$-\sqrt{3}$	4	-	incr.	$\cap$
	0	0	1	0	incr.	infl.
	$\pi/3$	$\sqrt{3}$	4	+	incr.	$\cup$



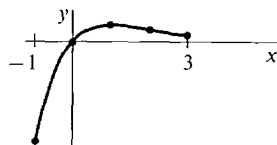
39	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	-2	$e^2$	$-e^2$	+	decr.	$\cup$
	0	1	-1	+	decr.	$\cup$
	2	$e^{-2}$	$-e^{-2}$	+	decr.	$\cup$



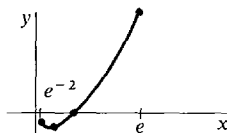
41	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$1/e$	-1	$e$	-	incr.	$\cap$
	1	0	1	-	incr.	$\cap$
	$e$	1	$1/e$	-	incr.	$\cap$



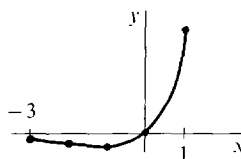
43	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	-1	$-e$	$2e$	-	incr.	$\cap$
	0	0	1	-	incr.	$\cap$
	1	$e^{-1}$	0	-	max.	$\cap$
	2	$2e^{-2}$	$-e^{-2}$	0	decr.	infl.
	3	$3e^{-3}$	$-2e^{-3}$	+	decr.	$\cup$



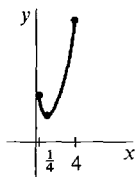
45	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$e^{-2}$	$-2e^{-2}$	-1	$e^2$	decr.	$\cup$
	$e^{-1}$	$-e^{-1}$	0	$e$	min.	$\cup$
	1	0	1	1	incr.	$\cup$
	$e$	$e$	2	$e^{-1}$	incr.	$\cup$



47	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	-3	$-3e^{-3}$	$-2e^{-3}$	-	decr.	$\cap$
	-2	$-2e^{-2}$	$-e^{-2}$	0	decr.	infl.
	-1	$-e^{-1}$	0	+	min.	$\cup$
	0	0	1	+	incr.	$\cup$
	1	$e$	$2e$	+	incr.	$\cup$



49	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\frac{1}{4}$	$4e^{\frac{1}{4}}$	$-12e^{\frac{1}{4}}$	+	decr.	$\cup$
	1	$e$	0	+	min.	$\cup$
	4	$e^4/4$	$\frac{3}{16}e^4$	+	incr.	$\cup$



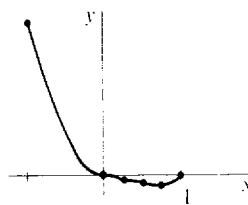
## Section 3.8

- 1  $f(0) = 1, f(1) = -1$     3  $f(4) = 0.236, f(9) = -2.838$   
 5  $f(0) = 0.586, f(1) = -0.732$     7  $f(0) = -1, f(1) = 1$     9  $f(0) = 1, f(1) = -1$   
 11  $f(0) = -1, f(1) = 1$     13  $f(0) = 0.9, f(\pi) = -1.1$     15  $f(1) = -1, f(e) = 0.632$   
 17 yes    19 yes    21 no    23 yes,  $f'(16/7) = 0$     25 no    27 yes    29 no  
 31 one    33 two    35  $1/2$     37  $8/27$     39 1    41  $\sqrt{3} - 1$   
 43  $f'(x) = 3x^2 - 3$  has no zeros in  $(-1, 1)$

## Extra Problems for Chapter 3

- 1  $A = 6V^{2/3}$     3  $x = 300t$     5  $s = 5x/4$     7  $16/\pi$  in./sec    9 0.01 in./sec  
 11 -3    13  $-1/4$     15  $-1 < x < 1$     17  $-2 < x < -1, 1 < x < 2$     19 none  
 21 max = 5 at  $x = 1$ , min = 3 at  $x = 2$     23 max = 4 at  $x = 0$ , min = 1 at  $x = \pm 1$   
 25 square of side  $\sqrt{2}$     27  $8b^3/27a^3$     29 min = -32 at  $x = -2$ , no max  
 31 base = height = 1    33 three zeros

33	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	-1	2	-7	+	decr.	$\cup$
	0	0	0		horiz.	infl.
	$\frac{1}{4}$	$-\frac{3}{256}$	$-\frac{1}{8}$	-	decr.	$\cap$
	$\frac{1}{2}$	$-\frac{1}{16}$	$-\frac{1}{4}$	0	decr.	infl.
	$\frac{3}{4}$	$-\frac{27}{256}$	0	+	min.	$\cup$
	1	0	1	+	incr.	$\cup$



- 37 none    39  $f(-1) = \sqrt[3]{7} - 2 < 0, f(0) = 1 > 0$

## Section 4.1

- 1 2    3  $5/4$     5  $11/8$     7 11    9  $-1/4$     11 136    13  $901/280 = 3.2$   
 15  $177/512 = 0.35$     17  $1 + \sqrt{2} + \sqrt{3}(\pi - 3) = 2.7$     19 1.90    21 1.55  
 23 3.18

## Section 4.2

- 1  $(16/3)x^{3/2}$     3  $t^3 + t$     5  $4t - t^3$     7  $-(7/2)s^{-2}$     9  $(1/3)(x-6)^3$   
 11  $(2/5)y^{5/2}$     13  $x^2/2, x \geq 0; -x^2/2, x < 0$     15  $2/3$     17 12    19 0  
 21  $32/3$     23 14    25 12    27 9.9    29  $16/3$     31  $1/6$     33 2

## Section 4.3

- 1  $x + x^2 + x^3 + C$     3  $(3/2)t^8 - (1/2)t^6 + (2/3)t^3 + t + C$     5  $(2/3)t^{3/2} + 2t^{1/2} + C$   
 7  $(1/6)(2x-3)^3 + C$     9  $(1/3)z^3 + 2z - z^{-1} + C$     11  $5 \sin x + C$   
 13  $x + \ln x + C$     15  $x + 2 \ln x - x^{-1} + C$     17  $12t - (4/3)t^{3/2} - t^2 + C$   
 19  $-4y^{-1} - 6y^{-1/2} + 2\sqrt{y} + C$     21  $(1/3)ax^3 + (1/2)bx^2 + cx + C$     23 0  
 25  $58/15$     27 0    29  $3 \ln 2$     31  $-\ln 3$     33  $(4/3)t^3 - t + 2$   
 35  $-2 \cos t + 12$     37  $v = (1/2)t^2, y = (1/6)t^3 + 1$   
 39  $v = t^3 + 1, y = (1/4)t^4 + t + 2$   
 41  $v = -(1/2)t^{-2} + 3/2, y = (1/2)t^{-1} + (3/2)t - 2$     43 (a), (d), (f), (g), (i), (l)  
 45  $y = t^3 + 2t + 1$

## Section 4.4

- 1  $-\frac{1}{2(2x+1)} + C$     3  $-\frac{1}{28}(3-4z)^7 + C$     5  $-(2/3)(1-t^2)^{3/2} + C$

- 7  $(1/30)(4 + 5x^2)^3 + C$     9  $-(1/3)\cos(3x) + C$     11  $-(3/2)\cos(4x - 1) + C$   
 13  $(1/2)\sin^2 \theta + C$     15  $-(1/4)\cos^4 \theta + C$     17  $-(1/2)\cos(x^2 + 1) + C$   
 19  $-\cos(\ln x) + C$     21  $(2/3)(\sin t)^{3/2} + C$     23  $(1/2)e^{2x} + C$   
 25  $ae^x - be^{-x} + C$     27  $\frac{1}{2}e^{x^2} + C$     29  $(b/a)e^{ax} + C$     31  $e^{\sin \theta} + C$   
 33  $\ln(x + 2) + C$     35  $\ln(e^x + 1) + C$     37  $x - \ln(x + 1) + C$   
 39  $2\ln(1 + \sqrt{x}) + C$     41  $-3t - 19\ln(5 - t) + C$     43  $(1/6)(x^4 + 5)^{3/2} + C$   
 45  $(1/3)(2 + y^2)^{3/2} + C$     47  $-(1 - u^2)^{1/2} + C$     49  $(2/3)(3s + 2)^{1/2} + C$   
 51  $-2(1 + x^{-1})^{1/2} + C$     53  $-(1/15)(3 + 5x^{-2})^{3/2} + C$     55  $-(2/3)(3 - \sqrt{x})^3 + C$   
 57  $-2z^{-1} - (1/2)z^{-2} + C$     59  $(2/3)(x^3 + 4)^{1/2} + C$     61  $(1/2)(1 + x^4)^{1/2} + C$   
 63  $(2/5)(t + 1)^{5/2} - (2/3)(t + 1)^{3/2} + C$     65  $(8/3)(1 - s)^{-3} - (1 - s)^{-2} + C$   
 67  $-(1/2)(y^2 + 1)^{-1} + (1/4)(y^2 + 1)^{-2} + C$   
 69  $(1/24)(4x + 1)^{3/2} - (1/8)(4x + 1)^{1/2} + C$  or  $(1/12)(2x - 1)(4x + 1)^{1/2} + C$   
 71  $-(2/27)(1 - 3u)^{3/2} + (2/45)(1 - 3u)^{5/2} + C$     73  $(1/6)(4x + 1)^{3/2} - (4x + 1)^{1/2} + C$   
 75  $-(1/4)\ln(1 - x^4) + C$     77  $(1/4)y^4 - (1/2)y^2 + (1/2)\ln(1 + y^2) + C$  or  
 $(1/4)(1 + y^2)^2 - (1 + y^2) + (1/2)\ln(1 + y^2) + C$   
 79  $(2/3)\ln(3u + 2) + 3/(3u + 2) + C$     81  $x - 4\sqrt{x} + 8\ln(2 + \sqrt{x}) + C$   
 83  $\ln(\sin \theta) + C$     85  $(1/b)\ln(a + bx) + C$     87  $-\ln(1 + \cos \theta) + C$   
 89  $(1/2)(\ln x)^2 + C$     91  $1/2$     93  $e - e^{-1}$     95  $\ln 2$  or  $(1/2)\ln 4$     97  $1/2$   
 99  $(2/3)(3\sqrt{3} - 1)$     101  $242/5$     103  $100$     105  $(1/6)(17\sqrt{17} - 27)$     107  $0$   
 109  $(\ln 7 - \ln 4)/3$     111  $2/3$     113  $93/35$     115  $0$     117  $f(g(x)) + C$

### Section 4.5

- 1  $56/3$     3  $2$     5  $(10\sqrt{5/3}) - 6$     7  $32/3$     9  $500/3$     11  $2$     13  $4$   
 15  $2$     17  $e^2 - 3$     19  $2(e - e^{-1})$     21  $2 - \ln 3$     23  $(3/2) - \ln 2$     25  $1/5$   
 27  $128\sqrt{6/5}$     29  $4/3$     31  $32/3$     33  $1/6$     35  $19/15$     37  $49/15$   
 39  $23/12$     41  $3/4$     43  $1 - (4/3) \cdot 2^{3/4} + (5/6) \cdot 2^{3/5} = 0.02$     45  $11/4$   
 47  $(3 + \sqrt{5})/2$     49  $\sqrt[3]{54}$  or  $3\sqrt[3]{2}$     51  $c = (1/2)^{2/3}$  or  $c = 2^{-2/3}$

### Section 4.6

- 1 In all cases, sum = 2, error = 0.  
 3 (a)  $\Delta x = 0.25$ : sum = 1.0426     $\Delta x = 0.1$ : sum = 1.0652  
 (b)  $\Delta x = 0.25$ : sum = 1.0594     $\Delta x = 0.1$ : sum = 1.0700  
 In all cases, there is no error estimate.  
 5 (a)  $\Delta x = 0.25$ : sum = 0.3229, error  $\leq 1/384$   
 $\Delta x = 0.1$ : sum = 0.3220, error  $\leq 1/2400$   
 (b)  $\Delta x = 0.25$ : sum = 0.3217108, error  $\leq 1/15,360$   
 $\Delta x = 0.1$ : sum = 0.3217505, error  $\leq 1/600,000$   
 7 (a)  $\Delta x = 0.5$ : sum = 2.8968, error  $\leq 1/48$   
 $\Delta x = 0.1$ : sum = 2.9012, error  $\leq 1/1200$   
 (b)  $\Delta x = 0.5$ : sum = 2.9013, error  $\leq 1/960$   
 $\Delta x = 0.1$ : sum = 2.901388, error  $\leq 1/600,000$   
 9 (a)  $\Delta x = 0.25$ : sum = 1.0968, error  $\leq \sqrt{2}/96$   
 $\Delta x = 0.1$ : sum = 1.0906, error  $\leq \sqrt{2}/600$   
 (b)  $\Delta x = 0.25$ : sum = 1.089413  
 $\Delta x = 0.1$ : sum = 1.089430  
 11 (a)  $\Delta x = 1$ : sum = 2.0214, error  $\leq 1$   
 $\Delta x = 0.1$ : sum = 1.9447, error  $\leq 0.01$   
 (b)  $\Delta x = 1$ : sum = 1.9587, error  $\leq 0.8$   
 $\Delta x = 0.1$ : sum = 1.945913, error  $\leq 0.0008$   
 13 (a)  $\Delta x = 3$ : sum = 1.8793, error  $\leq 45/8$   
 $\Delta x = 0.1$ : sum = 1.6685, error  $\leq 5/800$   
 (b)  $\Delta x = 3$ : sum = 1.738132  
 $\Delta x = 0.1$ : sum = 1.668231

- 15 (a)  $\Delta x = \pi/2$ : sum = 1.5708, error  $\leq \pi^3/48$  or 0.65  
 $\Delta x = \pi/10$ : sum = 1.9835, error  $\leq \pi^3/1200$  or 0.026  
 (b)  $\Delta x = \pi/2$ : sum = 2.0944, error  $\leq \pi^5/2880$  or 0.1  
 $\Delta x = \pi/10$ : sum = 2.00011, error  $\leq \pi^5/1,800,000$  or 0.00017
- 17 (a)  $\Delta x = 0.25$ : sum = 1.7272 error  $\leq e/192$  or 0.014  
 $\Delta x = 0.1$ : sum = 1.7197, error  $\leq e/1200$  or 0.002  
 (b)  $\Delta x = 0.25$ : sum = 1.718319, error  $\leq e/46,080$  or 0.00006  
 $\Delta x = 0.1$ : sum = 1.718283, error  $\leq e/1,800,000$  or 0.0000015
- 19 (a)  $\Delta x = 0.25$ : sum = 0.3837, error  $\leq 1/192$   
 $\Delta x = 0.1$ : sum = 0.3859, error  $\leq 1/1200$   
 (b)  $\Delta x = 0.25$ : sum = 0.386260, error  $\leq 1/46,080$   
 $\Delta x = 0.1$ : sum = 0.386293, error  $\leq 1/1,800,000$

### Extra Problems for Chapter 4

- 1 0.6025    3 7.875    5  $1/3$     7  $76 + 40\sqrt{5}$     9  $946\frac{2}{3}$     11 2  
 13  $2x + x^2/2 - x^3 + C$     15  $-(1/4)(x^2 - 1)^{-2} + C$     17  $-(1/3)(1 - 3u^2)^{1/2} + C$   
 19  $(1/3)[(2t + 1)^{3/2} - (2t - 1)^{3/2}] + C$     21  $(2/5)u^{5/2} - (4/3)u^{3/2} + C$   
 23  $2 \sin(x/2) + C$     25  $-e^{-t} + C$     27  $40/3$     29  $(e^4 - 1)/4$     31  $(x^3 + 2)^{1/2}$   
 33  $-\sqrt{u(u-1)^{1/2}}$     35  $(1/2)x^2 - x + b + 1/2$     37  $3x^2 - 6x + 5$   
 39  $F(x) = 1$  if  $x < 0$ ,  $(x^2/2) + 1$  if  $x \geq 0$     43  $\pi ab/2$     45  $xf(x) + \int_0^x f(t) dt$

### Section 5.1

- 1 3    3  $\infty$     5  $1/3$     7  $5/3$     9  $1/\sqrt{3}$     11  $\infty$     13  $\infty$     15 0  
 17  $-\infty$     19  $\infty$     21 0    23  $1/2$     25  $-1/4$     27 0    29  $-5/6$     31 0  
 33 does not exist    35  $\infty$     37 does not exist    39 does not exist    41  $\infty$   
 43 0    45  $-\infty$     47  $1/2$     49  $\infty$     51  $-3/2$     53 0    55 1    57 0  
 59 does not exist    61 0    63  $-\infty$     65 does not exist    67  $\infty$

### Section 5.2

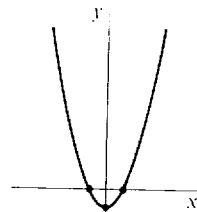
- 1  $1/6$     3  $1/16$     5 0    7  $-1/4$     9  $1/2$     11  $-1$     13 5    15  $\infty$   
 17 2    19 3    21 1    23 2    25 1    27 1    29 1    31 1    33  $1/2$   
 35 0    37  $1/2$     39  $1/2$     41  $4/\sqrt{2} = \sqrt{8}$     43 2    45  $\infty$     47  $3/2$   
 49 5    51 does not exist

### Section 5.3

- 1 (a) A, B, F, H (b) B, F, H (c) B, F (d) A, B, C, D, F, G, H  
 (e) B, D, F, G, H (f) B, D, F

3

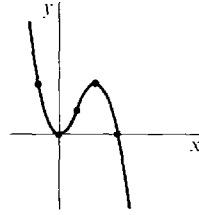
$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -x}$	$\infty$	$-\infty$			
0	0	-2	+	decr.	∪
1	-1	0	+	min.	∪
2	0	2	+	incr.	∪
$\lim_{x \rightarrow x}$	$\infty$	$\infty$			





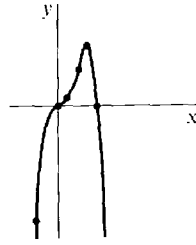
5

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	$\infty$	$-\infty$			
-2	$5\frac{1}{3}$	-6	+	decr.	$\cup$
0	0	0	+	min.	$\cup$
2	$2\frac{2}{3}$	2	0	incr.	infl.
4	$5\frac{1}{3}$	0	-	max.	$\cup$
6	0	-6	-	decr.	$\cup$
$\lim_{x \rightarrow \infty}$	$-\infty$	$-\infty$			



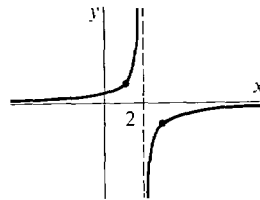
7

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	$-\infty$	$\infty$			
-2	-12	20	-	incr.	$\cup$
0	0	0	0	horiz.	infl.
1	$\frac{3}{4}$	2	+	incr.	$\cup$
2	4	4	0	incr.	infl.
3	$6\frac{3}{4}$	0	-	max.	$\cup$
4	0	-16	-	decr.	$\cup$
$\lim_{x \rightarrow \infty}$	$-\infty$	$-\infty$			



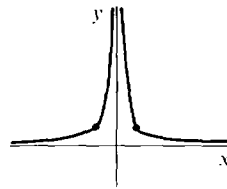
9

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	0	0			
1	1	1	+	incr.	$\cup$
$\lim_{x \rightarrow 2^-}$	$\infty$	$\infty$			
$\lim_{x \rightarrow 2^+}$	$-\infty$	$\infty$			
3	-1	1	-	incr.	$\cup$
$\lim_{x \rightarrow \infty}$	0	0			



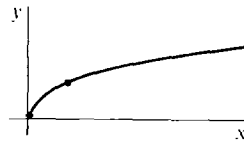
11

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	0	0			
-1	1	2	+	incr.	$\cup$
$\lim_{x \rightarrow 0^-}$	$\infty$	$\infty$			
$\lim_{x \rightarrow 0^+}$	$\infty$	$\infty$			
1	1	-2	+	decr.	$\cup$
$\lim_{x \rightarrow \infty}$	0	0			

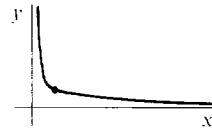


13

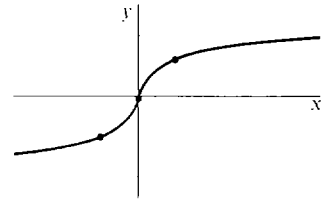
$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow 0^+}$	0	$\infty$			
1	1	$\frac{1}{2}$	-	incr.	$\cup$
$\lim_{x \rightarrow \infty}$	$\infty$	0			



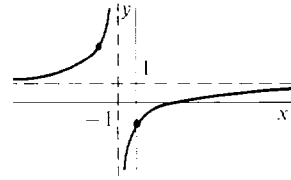
15	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow 0^+}$	$\infty$	$-\infty$			
	1	1	$-\frac{1}{2}$	+	decr.	$\cap$
	$\lim_{x \rightarrow \infty}$	0	0			



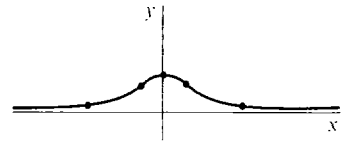
17	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow -\infty}$	$-\infty$	0			
	-1	-1	$\frac{1}{3}$	+	incr.	$\cup$
	$\lim_{x \rightarrow 0^-}$	0	$\infty$			
	$\lim_{x \rightarrow 0^+}$	0	$\infty$			
	1	1	$\frac{1}{3}$	-	incr.	$\cap$
	$\lim_{x \rightarrow \infty}$	$\infty$	0			



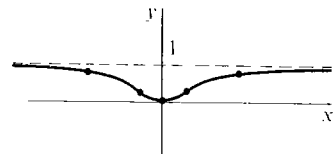
19	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow -\infty}$	1	0			
	-2	3	2	-	incr.	$\cap$
	$\lim_{x \rightarrow -1^-}$	$\infty$	$\infty$			
	$\lim_{x \rightarrow -1^+}$	$-\infty$	$\infty$			
	0	-1	2	-	incr.	$\cap$
	$\lim_{x \rightarrow \infty}$	1	0			



21	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow -\infty}$	0	0			
	-2	$\frac{1}{5}$	$\frac{4}{25}$	+	incr.	$\cup$
	$-1/\sqrt{3}$	$\frac{3}{4}$	1.9	0	incr.	$\cap$ infl.
	0	1	0	-	max.	$\cup$
	$1/\sqrt{3}$	$\frac{3}{4}$	-1.9	0	decr.	$\cap$ infl.
	2	$\frac{1}{5}$	$-\frac{4}{25}$	+	decr.	$\cup$
	$\lim_{x \rightarrow \infty}$	0	0			

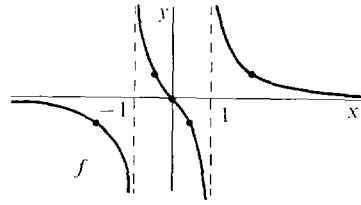


23	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow -\infty}$	1	0			
	-2	$\frac{4}{5}$	$-\frac{2}{25}$	-	decr.	$\cap$
	$-1/\sqrt{3}$	$\frac{1}{4}$	$-\sqrt{3}/2$	0	decr.	$\cap$ infl.
	0	0	0	+	min.	$\cup$
	$1/\sqrt{3}$	$\frac{1}{4}$	$\sqrt{3}/2$	0	incr.	$\cup$ infl.
	2	$\frac{4}{5}$	$\frac{2}{25}$	-	incr.	$\cap$
	$\lim_{x \rightarrow \infty}$	1	0			



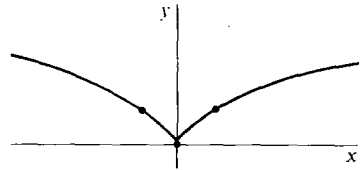
25

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	0	0			
-2	$-\frac{2}{3}$	$-\frac{5}{9}$	-	decr.	$\cap$
$\lim_{x \rightarrow -1^-}$	$-\infty$	$-\infty$			
$\lim_{x \rightarrow -1^+}$	$\infty$	$-\infty$			
$-\frac{1}{2}$	$\frac{2}{3}$	$-\frac{20}{9}$	+	decr.	$\cup$
0	0	-1	0	decr.	infl.
$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{20}{9}$	-	decr.	$\cap$
$\lim_{x \rightarrow 1^-}$	$-\infty$	$-\infty$			
$\lim_{x \rightarrow 1^+}$	$\infty$	$-\infty$			
2	$\frac{2}{3}$	$-\frac{5}{9}$	+	decr.	$\cup$
$\infty$	0	0			



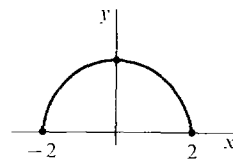
27

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	$\infty$	0			
-1	1	$-\frac{2}{3}$	-	decr.	$\cap$
$\lim_{x \rightarrow 0^-}$	0	$-\infty$		min. (cusp)	
$\lim_{x \rightarrow 0^+}$	0	$\infty$			
1	1	$\frac{2}{3}$	-	incr.	$\cup$
$\lim_{x \rightarrow \infty}$	$\infty$	0			



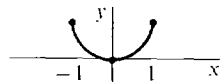
29

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -2^+}$	0	$\infty$			
0	2	0	-	max.	$\cap$
$\lim_{x \rightarrow 2^-}$	0	$-\infty$			



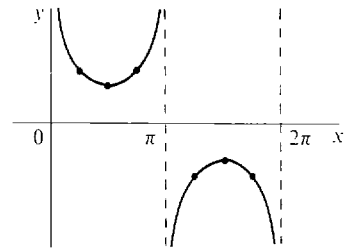
31

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -1^+}$	1	$-\infty$			
0	0	0	+	min.	$\cup$
$\lim_{x \rightarrow 1^-}$	1	$\infty$			



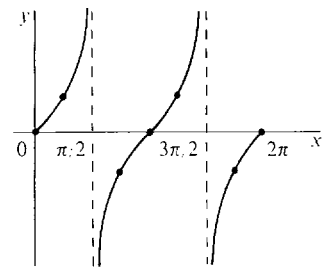
33

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow 0^+}$	$\infty$	$-\infty$			
$\pi/4$	$\sqrt{2}$	$-\sqrt{2}$	$+$	decr.	$\cup$
$\pi/2$	$1$	$0$	$+$	min.	$\cup$
$3\pi/4$	$\sqrt{2}$	$\sqrt{2}$	$+$	incr.	$\cup$
$\lim_{x \rightarrow \pi^-}$	$\infty$	$\infty$			
$\lim_{x \rightarrow \pi^+}$	$-\infty$	$\infty$			
$5\pi/4$	$-\sqrt{2}$	$\sqrt{2}$	$-$	incr.	$\cap$
$3\pi/2$	$-1$	$0$	$-$	max.	$\cap$
$7\pi/4$	$-\sqrt{2}$	$-\sqrt{2}$	$-$	decr.	$\cap$
$\lim_{x \rightarrow 2\pi}$	$-\infty$	$-\infty$			



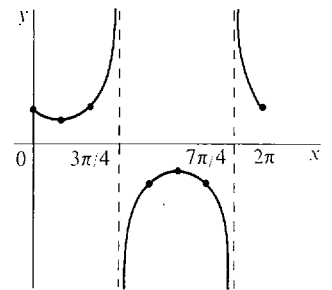
35

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$0$	$0$	$1$	$0$	incr.	infl.
$\pi/4$	$1$	$2$	$+$	incr.	$\cup$
$\lim_{x \rightarrow \pi/2^-}$	$\infty$	$\infty$			
$\lim_{x \rightarrow \pi/2^+}$	$-\infty$	$\infty$			
$3\pi/4$	$-1$	$2$	$-$	incr.	$\cap$
$\pi$	$0$	$1$	$0$	incr.	infl.
$5\pi/4$	$1$	$2$	$+$	incr.	$\cup$
$\lim_{x \rightarrow 3\pi/2^-}$	$\infty$	$\infty$			
$\lim_{x \rightarrow 3\pi/2^+}$	$-\infty$	$\infty$			
$7\pi/4$	$-1$	$2$	$-$	incr.	$\cap$
$2\pi$	$0$	$1$	$0$	incr.	infl.



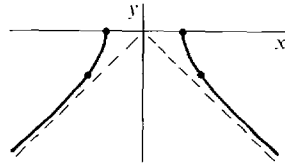
37

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$0$	$1$	$-1$	$+$	decr.	$\cup$
$\pi/4$	$\sqrt{2}/2$	$0$	$+$	min.	$\cup$
$\pi/2$	$1$	$1$	$+$	incr.	$\cup$
$\lim_{x \rightarrow 3\pi/4^-}$	$\infty$	$\infty$			
$\lim_{x \rightarrow 3\pi/4^+}$	$-\infty$	$\infty$			
$\pi$	$-1$	$1$	$-$	incr.	$\cap$
$5\pi/4$	$-\sqrt{2}/2$	$0$	$-$	max.	$\cap$
$3\pi/2$	$-1$	$-1$	$-$	decr.	$\cap$
$\lim_{x \rightarrow 7\pi/4^-}$	$-\infty$	$-\infty$			
$\lim_{x \rightarrow 7\pi/4^+}$	$\infty$	$-\infty$			
$2\pi$	$1$	$-1$	$+$	decr.	$\cup$



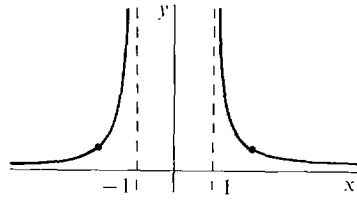
39

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	$-\infty$	1			
-3	$-\sqrt{5}$	$3/\sqrt{5}$	+	incr.	∪
$\lim_{x \rightarrow -2^-}$	0	$\infty$			
$\lim_{x \rightarrow 2^+}$	0	$-\infty$			
3	$-\sqrt{5}$	$-3/\sqrt{5}$	+	decr.	∪
$\lim_{x \rightarrow \infty}$	$-\infty$	-1			



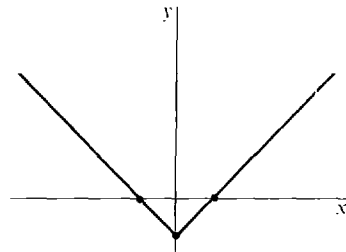
41

$x$	$f(x)$	$f(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	0	0			
-2	$1/\sqrt{3}$	$2/(3\sqrt{3})$	+	incr.	∪
$\lim_{x \rightarrow -1^-}$	$\infty$	$\infty$			
$\lim_{x \rightarrow 1^+}$	$\infty$	$-\infty$			
2	$1/\sqrt{3}$	$-2/(3\sqrt{3})$	+	decr.	∪
$\lim_{x \rightarrow \infty}$	0	0			



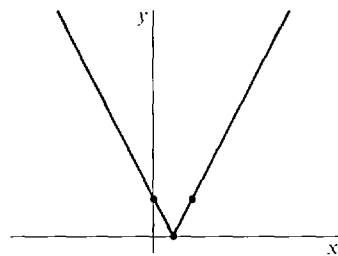
43

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	$\infty$	-1			
-1	0	-1	0	decr.	linear
$\lim_{x \rightarrow 0^-}$	-1	-1		min. (corner)	
$\lim_{x \rightarrow 0^+}$	-1	1			
1	0	1	0	incr.	linear
$\lim_{x \rightarrow \infty}$	$\infty$	1			



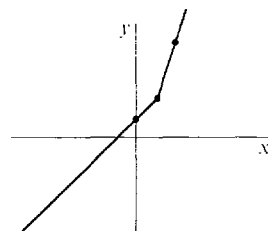
45

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	$\infty$	-2			
0	1	-2	0	decr.	linear
$\lim_{x \rightarrow \frac{1}{2}^-}$	0	-2		min. (corner)	
$\lim_{x \rightarrow \frac{1}{2}^+}$	0	2			
1	1	2	0	incr.	linear
$\lim_{x \rightarrow \infty}$	$\infty$	2			



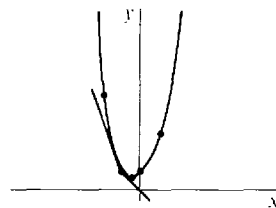
47

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	$-\infty$	1			
0	2	1	0	incr.	linear
$\lim_{x \rightarrow 2^-}$	4	1		corner	
$\lim_{x \rightarrow 2^+}$	4	3			
4	10	3	0	incr.	linear
$\lim_{x \rightarrow \infty}$	$\infty$	3			



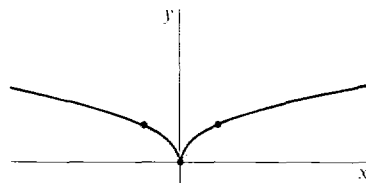
49

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	$\infty$	$-\infty$			
-2	5	-5	2	decr.	∪
$\lim_{x \rightarrow -1^-}$	1	-3		corner	
$\lim_{x \rightarrow -1^+}$	1	-1			
$-\frac{1}{2}$	$\frac{3}{4}$	0	2	min.	∪
0	1	1	2	incr.	∪
1	3	3	2	incr.	∪
$\lim_{x \rightarrow \infty}$	$\infty$	$\infty$			



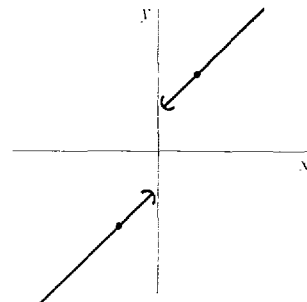
51

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	$\infty$	0			
-1	1	$-\frac{1}{2}$	-	decr.	∩
$\lim_{x \rightarrow 0^-}$	0	$-\infty$		min. (cusp)	
$\lim_{x \rightarrow 0^+}$	0	$\infty$			
1	1	$\frac{1}{2}$	-	incr.	∩
$\lim_{x \rightarrow \infty}$	$\infty$	0			

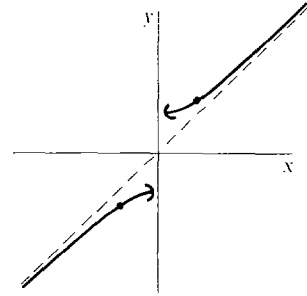


53

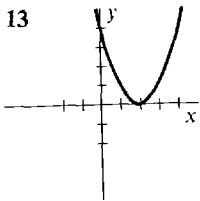
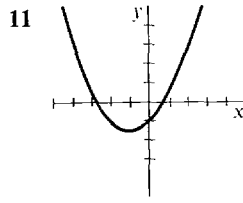
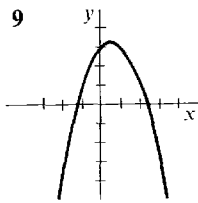
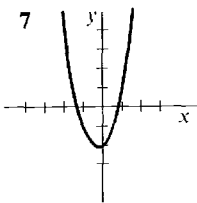
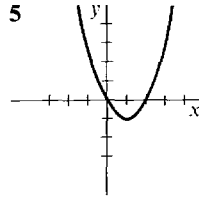
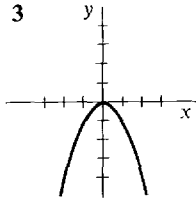
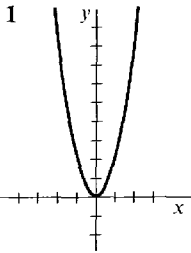
$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	$-\infty$	1			
-1	-2	1	0	incr.	linear
$\lim_{x \rightarrow 0^-}$	-1	1		jump	
$\lim_{x \rightarrow 0^+}$	1	1			
1	2	1	0	incr.	linear
$\lim_{x \rightarrow \infty}$	$\infty$	1			



55	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow -\infty}$	$-\infty$	1			
	$-1$	$-\sqrt{2}$	$1/\sqrt{2}$	$-$	incr.	$\cap$
	$\lim_{x \rightarrow 0^-}$	$-1$	0		jump	
	$\lim_{x \rightarrow 0^+}$	1	0			
	1	$\sqrt{2}$	$1/\sqrt{2}$	$+$	incr.	$\cup$
	$\lim_{x \rightarrow \infty}$	$\infty$	1			

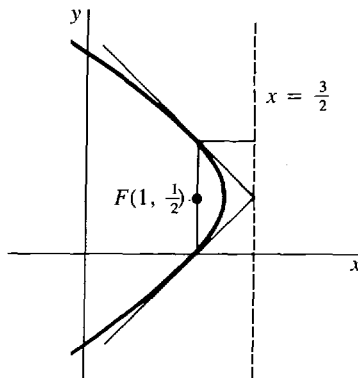
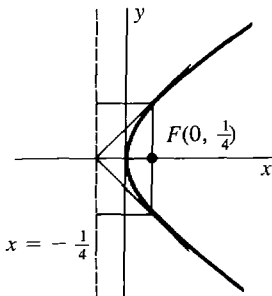


Section 5.4



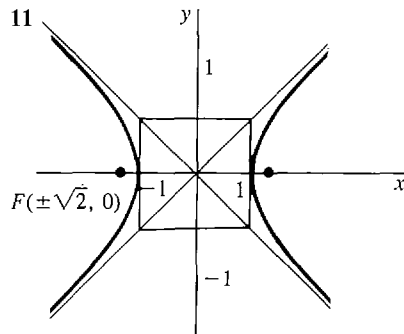
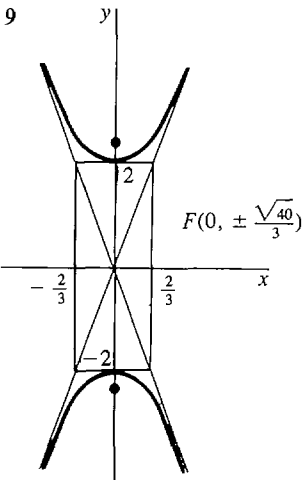
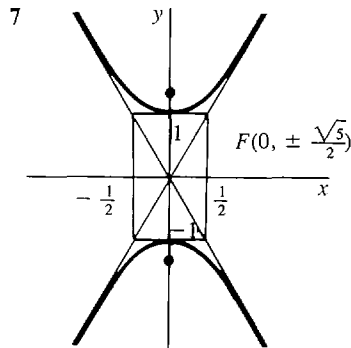
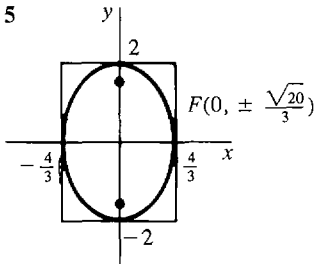
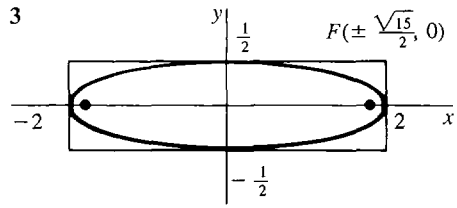
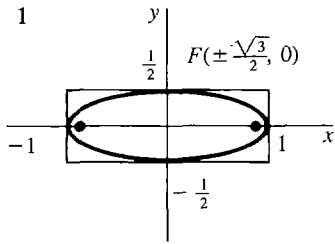
15 focus  $(1/4, 0)$ , directrix  $x = -1/4$

17 focus  $(3/2, 1/2)$ , directrix  $x = 2$



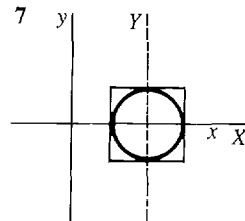
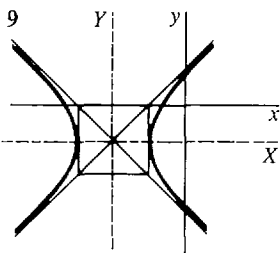
19  $y = (1/4)x^2 - x + 2$     21  $(1,3)$

Section 5.5

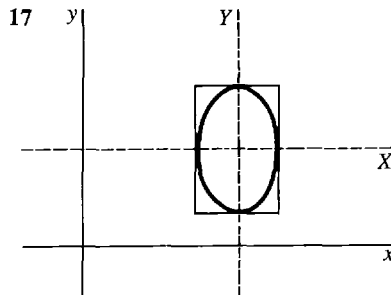
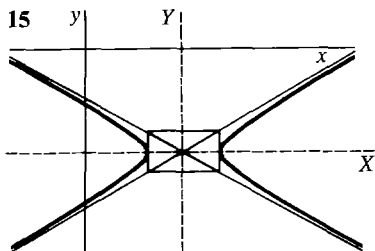
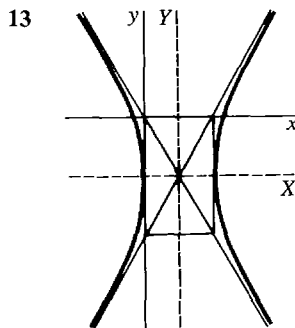
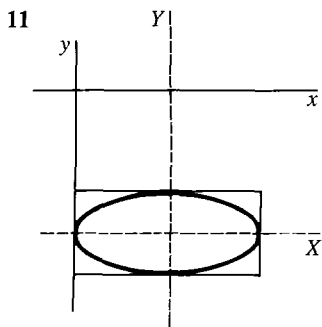


Section 5.6

1 hyperbola    3 parabola    5 ellipse







### Section 5.7

- 1  $\alpha = 45^\circ, X^2 - Y^2 + 8 = 0$     3  $\alpha = 45^\circ, -2X^2 + 6Y^2 = 1$   
 5  $\alpha = 30^\circ, 2X^2 - 2Y^2 = 7$     7  $\alpha = 22.5^\circ, 1.207X^2 - 0.207Y^2 = 3$   
 9  $\alpha = -15^\circ, 4.232X^2 + 0.768Y^2 = 5$

### Section 5.8

Any smaller value of  $\delta > 0$  or larger value of  $B$  is also correct.

- 1  $\delta = 10^{-3}$     3  $\delta = 2 - (0.51)^{-1} \sim 0.039$     5  $\delta = 10^{-4}$   
 7  $\delta = 1 - \sqrt{1 - 10^{-6}} \sim 5 \times 10^{-7}$     9  $\delta = 10^{-2}$     11  $\delta = 10^{-3}$     13  $\delta = 10^{-4}$   
 15  $\delta = 1 - \sqrt{0.99} \sim 5 \times 10^{-3}$     17  $B = 99/4 = 24.75$     19  $B = \frac{5 + \sqrt{8025}}{4} \sim 23.7$   
 21  $B = \frac{10^4 - 1}{5} \sim 2 \times 10^3$   
 23 For all  $\varepsilon > 0$  there exists  $\delta > 0$  such that whenever  $c - \delta < x < c$ ,  $|f(x) - L| < \varepsilon$ .  
 25 For all  $A > 0$  there exists  $B > 0$  such that whenever  $x > B$ ,  $f(x) < -A$ .

### Section 5.9

- 1 1.42332    3 0.63688    5 1.22074    7 2.93923    9 0.38197    11 0.56714  
 13 1.10606    15 1.30633

### Section 5.10

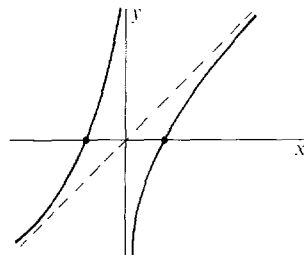
- 1  $f'(1) = 2$ , error  $\leq \Delta x$     3  $f'(4) = -(1/8)$ , error  $\leq (3/128)\Delta x$   
 5  $f'(3) = -1/9$ , error  $\leq (1/27)\Delta x$     7  $f'(0) = 1$ , error  $\leq 0.5\Delta x$   
 9  $f'(\pi/3) = -\sqrt{3}$ , error  $\leq 2\Delta x$     11  $f'(1) = 1$ , error  $\leq \Delta x/2$

- 13  $f'(1) = e$ , error  $\leq (1/2)e \Delta x$   
 15  $f'(100) = 1/20$ , error  $\leq (1/8) \cdot 99^{-3/2} \Delta x$  or  $0.000127 \Delta x$   
 17  $f'(2) = (2/\sqrt{5})$ , error  $\leq 1/(4\sqrt{2}) \Delta x$     19  $f'(1) = \sqrt{2} + 2^{-3 \cdot 2}$ , error  $\leq \Delta x/2$   
 21  $\sqrt{65} \sim 8\frac{1}{16}$ , error  $\leq 1/4096$     23  $(0.301)^4$  or  $0.008208$ , error  $\leq 6 \times 10^{-7}$   
 25  $1/97$  or  $10^{-2} + 3 \times 10^{-4} = 0.0103$ , error  $\leq 10^{-5}$   
 27  $\sqrt{1.02} + \sqrt[3]{1.02} \sim 2\frac{1}{60}$ , error  $\leq \frac{17}{8} \times 10^{-4}$     29  $(1.003)^5$  or  $1.015$ , error  $\leq 10^{-4}$   
 31  $\sin(\pi/3 + 0.004)$  or  $\sqrt{3}/2 + 0.002$ , error  $\leq 8 \times 10^{-6}$   
 33  $\tan(0.005)$  or  $0.005$ , error  $\leq 10^{-6}$     35  $e^{0.002}$  or  $1.002$ , error  $\leq 3 \times 10^{-6}$   
 37  $\ln(1.006)$  or  $0.006$ , error  $\leq 18 \times 10^{-6}$

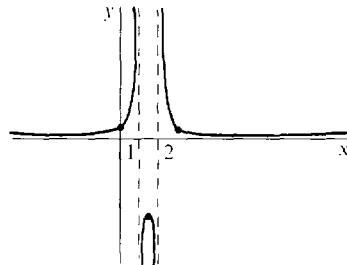
Extra Problems for Chapter 5

1 0 3 0 5  $\infty$  7 0 9 3/2

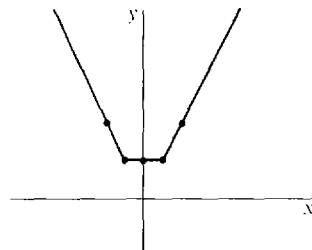
11	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow -\infty}$	$-\infty$	1			
	-1	0	2	+	incr.	∪
	$\lim_{x \rightarrow 0^-}$	$\infty$	$\infty$			
	$\lim_{x \rightarrow 0^+}$	$-\infty$	$\infty$			
	1	0	2	-	incr.	∩
	$\lim_{x \rightarrow \infty}$	$\infty$	1			

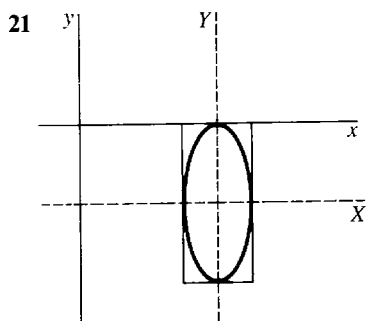
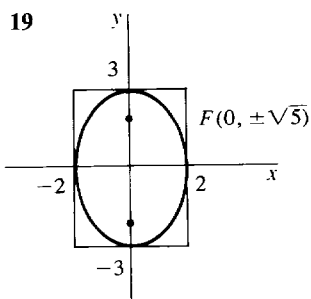
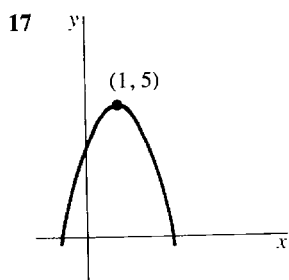


13	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow -\infty}$	0	0			
	0	$\frac{1}{2}$	$\frac{3}{4}$	+	incr.	∪
	$\lim_{x \rightarrow 1^-}$	$\infty$	$\infty$			
	$\lim_{x \rightarrow 1^+}$	$-\infty$	$\infty$			
	$\frac{3}{2}$	-4	0	-	max.	∩
	$\lim_{x \rightarrow 2^-}$	$-\infty$	$-\infty$			
	$\lim_{x \rightarrow 2^+}$	$\infty$	$-\infty$			
	3	$\frac{1}{2}$	$-\frac{3}{4}$	+	decr.	∪
	$\lim_{x \rightarrow \infty}$	0	0			



15	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow -\infty}$	$\infty$	-2			
	-2	4	-2	0	decr.	linear
	$\lim_{x \rightarrow -1^-}$	2	-2		corner	
	$\lim_{x \rightarrow -1^+}$	2	0			
	0	2	0	0	constant	linear
	$\lim_{x \rightarrow 1^-}$	2	0		corner	
	$\lim_{x \rightarrow 1^+}$	2	2			
	2	4	2	0	incr.	linear
	$\lim_{x \rightarrow \infty}$	$\infty$	2			





- 23  $\alpha = 45^\circ$ ,  $X^2 - Y^2 = 18$     25  $\delta \leq 0.155$     27 1.7866    29  $(3/4096) \Delta x$   
 31  $25 - 2/15 = 24.8666$ , error  $\leq 1/(9 \cdot (124)^{4/3})$

### Section 6.1

- 1  $1/3$     3  $\sqrt{3}/12$     5  $\pi/24$     7  $2cr^2/3$

### Section 6.2

- 1 (a)  $\pi/5$  (b)  $\pi/2$     3 (a)  $8\pi$  (b)  $128\pi/5$     5 (a)  $\pi/3$  (b)  $\pi/3$   
 7 (a)  $4\pi/3$  (b)  $(2/3)\pi(2\sqrt{2} - 1)$     9 (a)  $31\pi/160$  (b)  $\pi$     11 (a)  $4\pi/45$  (b)  $\pi/3$   
 13 (a)  $27\pi$  (b)  $18\pi$     15 (a)  $2\pi/35$  (b)  $\pi/10$     17 (a)  $\pi/3$  (b)  $\pi/9$   
 19  $20\pi/3$  (b)  $29\pi/6$     21 (a)  $16\pi/3$  (b)  $184\pi/15$     23  $2\pi$     25  $(\pi^2/4) - (\pi/2)$   
 27  $(\pi/2)(e^2 - 1)$     29  $(\pi/6)(e^2 - 1)$     31  $\pi \ln 2$     33  $\pi(3 - \ln 4)$     35  $2\pi$   
 37  $2\pi$     39  $\pi(e - 1)$     41  $2\pi(e^{-1} - e^{-4})$     43  $2\pi \ln 2$     45  $(\pi/2) \ln 7$   
 47  $\frac{4}{3}\pi(r^2 - a^2)^{3/2}$     49  $\frac{1}{3}\pi r^2 h - \pi a^2 h + \frac{2\pi h a^3}{3r}$   
 51 (a)  $\pi\left(\frac{b^2 a}{3} + \frac{2}{3}r^3 - r^2 a + \frac{a^3}{3}\right)$  or  $\frac{2}{3}\pi r^2(r - a)$   
 (b)  $\frac{2}{3}\pi(ba^2 + b^3)$  or  $\frac{2}{3}\pi[a^2\sqrt{r^2 - a^2} + (r^2 - a^2)^{3/2}]$     53  $2\pi^2 cr^2$

### Section 6.3

- 1  $\frac{2}{3}(6\sqrt{6} - 3\sqrt{3})$     3  $\frac{8}{3}(3\sqrt{3}/2\sqrt{2} - 1)$     5  $\frac{8}{27}(10\sqrt{10} - 1)$     7  $379/12$   
 9  $387/20$     11  $3/2$     13  $36$     15  $\frac{4}{3}(2^{5/2} - 1)$     17  $2\sqrt{3}/3$   
 19  $8 \cdot 10\sqrt{10} - 37\sqrt{37}$     21  $2\pi$     23  $4\sqrt{2}$     25  $\frac{t^2}{\sqrt{1 + 4t^2}}$   
 27  $\int_0^4 \sqrt{1 + (4x - 1)^2} dx$     29  $\int_1^2 \sqrt{4 + \frac{1}{4t}} dt$

$$31 \int_1^2 \sqrt{1 + \frac{1}{4x}} dx \sim \frac{1}{4} \left( \frac{\sqrt{5}}{4} + \sqrt{\frac{3}{2}} + \sqrt{\frac{7}{4}} + \sqrt{2} + \frac{3}{4} \right) \sim 1.32$$

$$33 \int_1^5 \sqrt{1+x^{-4}} dx \sim \frac{\sqrt{2}}{2} + \frac{\sqrt{17}}{4} + \frac{\sqrt{82}}{9} + \frac{\sqrt{257}}{16} + \frac{\sqrt{626}}{50} \sim 4.246$$

## Section 6.4

$$1 \frac{\pi}{6}(17\sqrt{17} - 1) \quad 3 \frac{2\pi}{81} \left( \frac{2}{5} \cdot 10^{5/2} - \frac{2}{3} \cdot 10^{3/2} + \frac{4}{15} \right) \sim 8.2 \quad 5 \quad 256\pi/15$$

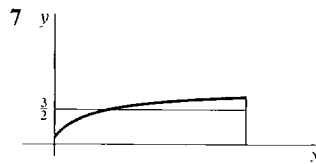
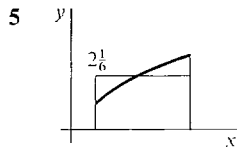
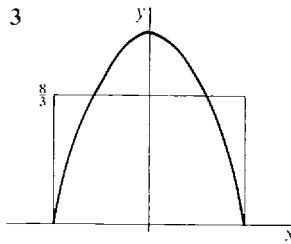
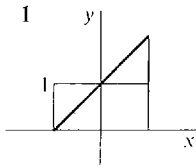
$$7 \quad 1575\pi/8 \quad 9 \quad \frac{2\pi}{3}(10\sqrt{10} - 2\sqrt{2}) \quad 11 \quad \pi(2\sqrt{2} - 1) \quad 13 \quad \frac{\pi}{9}(2\sqrt{2} - 1)$$

$$15 \quad 47\pi/16 \quad 17 \quad 7\pi/9 \quad 19 \quad 16\pi\sqrt{5} \quad 21 \quad 2\pi ra \quad 23 \quad (a) \int_0^1 2\pi x \sqrt{1+25x^8} dx$$

$$(b) \int_0^1 2\pi x^5 \sqrt{1+25x^8} dx \quad 25 \quad (a) \int_1^{10} 2\pi(t^2+t)\sqrt{8t^2+4t+1} dt$$

$$(b) \int_1^{10} 2\pi(t^2-1)\sqrt{8t^2+4t+1} dt \quad 27 \quad \int_0^1 \pi x^2 \sqrt{1+x^2} dx \sim 1.357$$

## Section 6.5



$$9 \quad 3 - (2/\sqrt{3}) \quad 11 \quad -6 \quad 13 \quad 0 \quad 15 \quad 2/\pi \quad 17 \quad 1/\pi \quad 19 \quad (1/2)(e - e^{-1})$$

$$21 \quad (1/3)\ln 4 \quad 23 \quad 1 \quad 25 \quad 8/9 \quad 27 \quad 6\sqrt{15}/25 \quad 29 \quad 13/2 \quad 31 \quad 116/45$$

$$33 \quad (a) \quad 2 \quad (b) \quad 32/7$$

## Section 6.6

$$1 \quad (a) \quad 4 \quad (b) \quad M_x = 0, M_y = 4 \quad (c) \quad (1, 0) \quad 3 \quad (a) \quad 128/3 \quad (b) \quad M_x = 512/5, M_y = 0$$

$$(c) \quad (0, 12/5) \quad 5 \quad (a) \quad 6k \quad (b) \quad M_x = 8k, M_y = 2k \quad (c) \quad (\frac{1}{3}, \frac{4}{3}) \quad 7 \quad (a) \quad 128/15$$

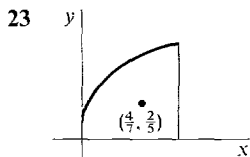
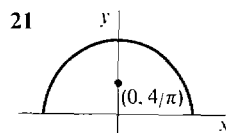
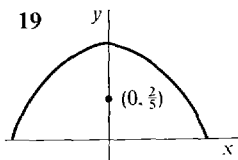
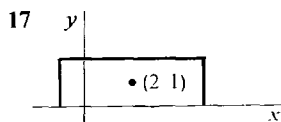
$$(b) \quad M_x = 1024/105, M_y = 0 \quad (c) \quad (0, \frac{8}{9}) \quad 9 \quad (a) \quad \frac{1}{2} \quad (b) \quad M_x = \frac{1}{8}, M_y = \frac{1}{3} \quad (c) \quad (\frac{2}{3}, \frac{1}{4})$$

$$11 \quad (a) \quad 2\sqrt{2} - 2 \quad (b) \quad M_x = 1 - \frac{\sqrt{2}}{2}, M_y = \frac{4\sqrt{2} - 2}{3}$$

$$(c) \quad \left( \frac{2\sqrt{2} - 1}{3\sqrt{2} - 3}, \frac{2 - \sqrt{2}}{4\sqrt{2} - 4} \right) \text{ or } \left( 1 + \frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{4} \right)$$

$$13 \quad (a) \quad 4/3 \quad (b) \quad M_x = 1/2, M_y = 0 \quad (c) \quad (0, 3/8)$$

$$15 \quad (a) \quad 128/3 \quad (b) \quad M_x = 1024/15, M_y = 512/5 \quad (c) \quad (\frac{12}{5}, \frac{8}{3})$$



- 25  $2/3$     27  $e^2 - 1$     29  $(2 \ln 2, 7/24)$     31  $(8/15, 16/105)$     33  $(1/2, 2/5)$   
 35 80 ft lbs    37  $320/3$  ft lbs    39 50 ft lbs    41 215 ft lbs    43  $9k/400$

### Section 6.7

- 1  $1/2$     3 diverges to  $\infty$     5 diverges to  $-\infty$     7 diverges to  $\infty$     9  $1/2$   
 11 diverges to  $\infty$     13 3    15 6    17  $-3/2$     19 3    21 diverges to  $\infty$   
 23 diverges to  $\infty$     25 diverges    27  $1/2$     29 diverges    31  $4\sqrt{2}$     33  $\infty$   
 37 4    39  $2/3$     43 (a)  $\pi$  (b)  $\infty$     45 (a)  $\infty$  (b)  $8\pi$     47  $4/3$   
 49 (a)  $11\pi/9$  (b)  $16\pi/15$     51 (a)  $\frac{\pi}{6}(5\sqrt{5} - 1)$  (b)  $\int_0^1 2\pi x \sqrt{1 + \frac{1}{4x}} dx$   
 53 (a)  $2\pi r a$  (b)  $2\pi r^2 - 2\pi r \sqrt{r^2 - a^2}$

### Extra Problems for Chapter 6

- 1  $8r^3/3$     3  $2r^3/3$     5 (a)  $11\pi/3$  (b)  $(16\pi/3) - 2\pi\sqrt{3}$     7 (a)  $16\pi$  (b)  $16\pi/3$   
 9  $\pi p/(p+1)$     11  $\frac{1}{27}(46\sqrt{46} - 10\sqrt{10})$     13 5    15  $(b-a)\sqrt{A^2 + C^2}$   
 17  $\frac{\pi}{3A} [(8A^2 + 4AB + B^2)^{3/2} - (4A^2 + B^2)^{3/2}]$     19  $\frac{b^{p+1} - 1}{(b-1)(p+1)}$     21  $(p+1)^{-1/p}$   
 23  $(\frac{1}{2}, \frac{2}{3})$     25  $(\frac{2}{3}, 0)$     27 15 ft lbs    29 diverges to  $\infty$     31  $-5/4$   
 33 diverges to  $\infty$     35 8    37  $a^2 h/3$     39  $m = \pi\rho, M_x = 2\rho, M_y = 0, (\bar{x}, \bar{y}) = (0, 2/\pi)$   
 43  $\pi\rho r^4/4$     45  $\omega\rho b^2/2$

### Section 7.1

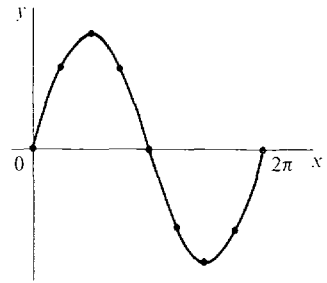
- 7  $(\pi/4) + k\pi$     9 none    13 0    15 0    17 0    19  $k\pi$

### Section 7.2

- 1  $5 \cos(5x)$     3  $6\theta \cos(3\theta^2)$     5  $4 \sec^2(4\theta - 3)$     7  $a \cos \theta - b \sin \theta$   
 9  $-\frac{1}{2\sqrt{x}} \sin(\sqrt{x})$     11  $\cos \theta \sec^2(\sin \theta)$     13  $\frac{3 \csc(3t) \cot(3t)}{(2 + \csc(3t))^2}$     15  $\frac{1}{2 \sin y \cos y}$   
 17  $3/2$     19  $\infty$     21 2    23 1

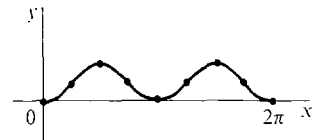
25

$x$	$f(x)$	$f'(x)$	$f''(x)$		
0	0	3	0	incr.	infl.
$\pi/4$	$3/\sqrt{2}$	$3/\sqrt{2}$	-	incr.	$\cap$
$\pi/2$	3	0	-	max.	$\cap$
$3\pi/4$	$3/\sqrt{2}$	$-3/\sqrt{2}$	-	decr.	$\cap$
$\pi$	0	-3	0	decr.	infl.
$5\pi/4$	$-3/\sqrt{2}$	$-3/\sqrt{2}$	+	decr.	$\cup$
$3\pi/2$	-3	0	+	min.	$\cup$
$7\pi/4$	$-3/\sqrt{2}$	$3/\sqrt{2}$	+	incr.	$\cup$
$2\pi$	0	3	0	incr.	infl.



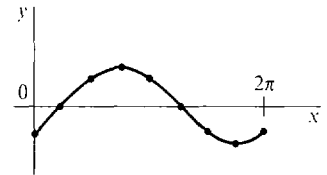
27

$x$	$f(x)$	$f'(x)$	$f''(x)$		
0	0	0	+	min.	$\cup$
$\pi/4$	1/2	1	0	incr.	infl.
$\pi/2$	1	0	-	max.	$\cap$
$3\pi/4$	1/2	-1	0	decr.	infl.
$\pi$	0	0	+	min.	$\cup$
$5\pi/4$	1/2	1	0	incr.	infl.
$3\pi/2$	1	0	-	max.	$\cap$
$7\pi/4$	1/2	-1	0	decr.	infl.
$2\pi$	0	0	+	min.	$\cup$



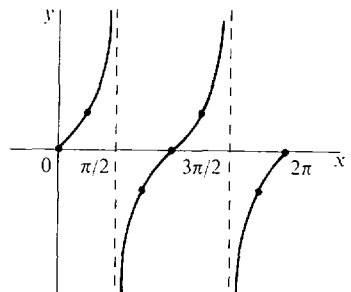
29

$x$	$f(x)$	$f'(x)$	$f''(x)$		
0	$-1/\sqrt{2}$	$1/\sqrt{2}$	+	incr.	$\cup$
$\pi/4$	0	1	0	incr.	infl.
$\pi/2$	$1/\sqrt{2}$	$1/\sqrt{2}$	-	incr.	$\cap$
$3\pi/4$	1	0	-	max.	$\cap$
$\pi$	$1/\sqrt{2}$	$-1/\sqrt{2}$	-	decr.	$\cap$
$5\pi/4$	0	-1	0	decr.	infl.
$3\pi/2$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	+	decr.	$\cup$
$7\pi/4$	-1	0	+	min.	$\cup$
$2\pi$	$-1/\sqrt{2}$	$1/\sqrt{2}$	+	incr.	$\cup$

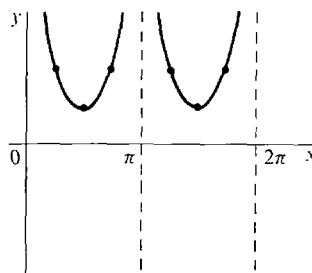


31

$x$	$f(x)$	$f'(x)$	$f''(x)$		
0	0	1	0	incr.	infl.
$\pi/4$	1	2	+	incr.	$\cup$
$\lim_{x \rightarrow \pi/2^-}$	$\infty$	$\infty$			
$\lim_{x \rightarrow \pi/2^+}$	$-\infty$	$\infty$			
$3\pi/4$	-1	2	-	incr.	$\cap$
$\pi$	0	1	0	incr.	infl.
$5\pi/4$	1	2	+	incr.	$\cup$
$\lim_{x \rightarrow 3\pi/2^-}$	$\infty$	$\infty$			
$\lim_{x \rightarrow 3\pi/2^+}$	$-\infty$	$\infty$			
$7\pi/4$	-1	2	-	incr.	$\cap$
$2\pi$	0	1	0	incr.	infl.



33	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow 0^+}$	$\infty$	$-\infty$			
	$\pi/4$	2	-4	+	decr.	∪
	$\pi/2$	1	0	+	min.	∪
	$3\pi/4$	2	4	+	incr.	∪
	$\lim_{x \rightarrow \pi^-}$	$\infty$	$\infty$			
	$\lim_{x \rightarrow \pi^+}$	$\infty$	$-\infty$			
	$5\pi/4$	2	-4	+	decr.	∪
	$3\pi/2$	1	0	+	min.	∪
	$7\pi/4$	2	4	+	incr.	∪
	$\lim_{x \rightarrow 2\pi^-}$	$\infty$	$\infty$			



37  $-\frac{1}{2} \cos(2t) + C$     39  $\frac{1}{3} \sec^3 x + C$     41  $2 \sin(\sqrt{x}) + C$     43  $-\frac{1}{5} \csc(5\theta) + C$   
 45  $2\sqrt{\sec x} + 1 + C$     47  $\tan \theta - \sec \theta + C$     49 2    51  $(3 - \sqrt{3})/2$     53  $\infty$   
 55  $-\frac{3}{10}$  radians/sec    57 6    59  $1/2$     61  $2\pi$     63  $\sqrt{2}$     65  $\pi\sqrt{2}$

## Section 7.3

1  $\pi/3$     3  $-\pi/4$     5  $\pi/3$     7  $\sqrt{1-x^2}$     9  $\frac{\pi}{2} - x$   
 15  $\frac{5}{|5x-2|\sqrt{(5x-2)^2-1}}$     17  $\frac{2x}{\sqrt{1-x^4}}$     19  $\arcsin t + \frac{t}{\sqrt{1-t^2}}$     21  $\frac{1-x}{\sqrt{1-x^2}}$   
 23  $\arcsin x$     25  $\frac{1}{2\sqrt{x(x+1)}}$     27  $-\pi/2$     29 1    31  $\frac{1}{3} \arctan(x/3) + C$   
 33  $\arcsin(2x-1) + C$     35  $\operatorname{arcsec}(2x) + C$     37  $\frac{1}{2} \arcsin(x^2) + C$   
 39  $2 \operatorname{arcsec}(\sqrt{x}) + C$  or  $2 \arctan(\sqrt{x-1}) + C$     41  $\frac{1}{2} (\arcsin x)^2 + C$     43  $\pi/3$   
 45  $\pi/6$     47  $\pi/a$     49  $\pi/2$

## Section 7.4

1  $x \sin x + \cos x + C$     3  $t^2 \sin t + 2t \cos t - 2 \sin t + C$   
 5  $-\frac{1}{2} t \cos(2t-1) + \frac{1}{4} \sin(2t-1) + C$   
 7  $-\frac{x^2}{4} \cos(4x) + \frac{x}{8} \sin(4x) + \frac{1}{32} \cos(4x) + C$   
 9  $\frac{1}{4} x^4 \operatorname{arcsec} x - \frac{1}{12} (x^2-1)^{3/2} - \frac{1}{4} \sqrt{x^2-1} + C$   
 11  $-2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x}) + C$     13  $(x+1) \arctan(\sqrt{x}) - \sqrt{x} + C$   
 15  $x^2 \sqrt{x^2-1} - \frac{2}{3} (x^2-1)^{3/2} + C$  or  $\frac{1}{3} (x^2-1)^{3/2} + \sqrt{x^2-1} + C$  or  $(\frac{1}{3} x^2 + \frac{2}{3}) \sqrt{x^2-1} + C$   
 17  $-\frac{1}{4} x \cos(2x) + \frac{1}{8} \sin(2x) + C$  or  $\frac{1}{2} x \sin^2 x - \frac{1}{4} x + \frac{1}{4} \sin x \cos x + C$   
 19  $\frac{2}{3} \sin^3 \theta + C$  or  $\frac{1}{3} [\cos \theta \sin(2\theta) - 2 \sin \theta (\cos(2\theta))] + C$   
 21  $\frac{1}{8} \cos(4x) - \frac{1}{12} \cos(6x) + C$  or  $\frac{5}{24} \sin x \sin(5x) + \frac{1}{24} \cos x \cos(5x) + C$   
 23  $\frac{1}{2} \sin(t^2) - \frac{1}{2} t^2 \cos(t^2) + C$     25  $\frac{1}{x} \cos\left(\frac{1}{x}\right) - \sin\left(\frac{1}{x}\right) + C$   
 27  $(t^2+4)^{3/2} \left(\frac{t^2}{5} - \frac{8}{15}\right) + C$  or  $\frac{1}{3} t^2 (t^2+4)^{3/2} - \frac{2}{15} (t^2+4)^{5/2} + C$  or  $-\frac{4}{3} (t^2+4)^{3/2} + \frac{1}{5} (t^2+4)^{5/2} + C$     29  $(\pi/2) - 1$     31  $\pi/2$     33  $1/2$   
 35  $\frac{2}{3} \pi - \frac{\sqrt{3}}{2}$

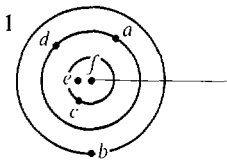
## Section 7.5

- 1  $(1/\cos t) + \cos t + C$     3  $-\cot x - x + C$     5  $\frac{1}{4} \cos^3 x \sin x + \frac{3}{8} \sin x \cos x + \frac{3}{8} x + C$   
 or  $\frac{3}{8} x + \frac{1}{4} \sin(2x) + \frac{1}{32} \sin(4x) + C$     7  $\frac{1}{6} \tan^6 x + \frac{1}{4} \tan^4 x + C$  or  
 $\frac{1}{6} \sec^6 x - \frac{1}{4} \sec^4 x + C$     9  $\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$     11  $-\frac{1}{3} \cot^3 \theta + C$   
 13  $\frac{2}{9} (\tan x)^{9/2} + \frac{2}{5} (\tan x)^{5/2} + C$     15  $\tan \theta - \cot \theta + C$  or  $-2 \cot(2\theta) + C$   
 17  $-\cot \theta + \csc \theta + C$     19  $4/3$     21  $\frac{\pi}{4} - \frac{2}{3}$     23  $3\pi/8$   
 25  $\cos(\sqrt{x}) \sin(\sqrt{x}) + \sqrt{x} + C$  or  $\frac{1}{2} \sin(2\sqrt{x}) + \sqrt{x} + C$   
 27  $-x \cos x + \frac{1}{3} x \cos^3 x + \frac{2}{3} \sin x + \frac{1}{9} \sin^3 x + C$   
 29  $\frac{1}{3} x \sin^3 x + \frac{1}{3} \cos x - \frac{1}{9} \cos^3 x + C$     31  $\frac{1}{9} \tan^9 \theta + \frac{2}{7} \tan^7 \theta + \frac{1}{5} \tan^5 \theta + C$   
 33  $\frac{1}{2} \sec x \tan x + \frac{1}{2} \int \sec x dx$     35  $\frac{1}{2} \sec x \tan x - \frac{1}{2} \int \sec x dx$   
 37  $-\frac{1}{4} \csc^4 x \cos x + \frac{1}{8} \csc^2 x \cos x - \frac{1}{8} \int \csc x dx$     39  $\int \sec x dx + \int \csc x dx$   
 41  $\int x^n \sin x dx = -x^n \cos x + n \int x^{n-1} \cos x dx = -x^n \cos x + nx^{n-1} \sin x - n(n-1) \int x^{n-2} \sin x dx$   
 43 (a)  $\pi^2/16$  (b)  $\pi^2/4$

## Section 7.6

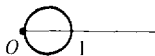
- 1  $\frac{1}{2} \arcsin(2x) + C$     3  $\frac{1}{3}(9+x^2)^{3/2} - 9(9+x^2)^{1/2} + C$  or  $\sqrt{9+x^2}(\frac{1}{3}x^2 - 6) + C$   
 5  $\frac{x}{4\sqrt{4-x^2}} + C$     7  $\arccos\left(\frac{\cos \theta}{\sqrt{2}}\right) + C$  or  $-\arcsin\left(\frac{\cos \theta}{\sqrt{2}}\right) + C$   
 9  $-\frac{1}{x} - \arctan x + C$     11  $2 \arcsin(x/2) - \frac{1}{2}x\sqrt{4-x^2} + C$   
 13  $\frac{1}{8} \arcsin x - \frac{1}{4}x(1-x^2)^{3/2} + \frac{1}{8}x\sqrt{1-x^2} + C$   
 15  $2 \arcsin\left(\frac{x-2}{2}\right) + \frac{1}{2}(x-2)\sqrt{4x-x^2} + C$     17  $\frac{1}{48}(4x^2-1)^{3/2} + \frac{1}{16}\sqrt{4x^2-1} + C$   
 or  $\sqrt{4x^2-1}\left(\frac{1}{24} - \frac{1}{2}x^2\right) + C$     19  $\frac{1}{3}(a^2-x^2)^{3/2} - a^2\sqrt{a^2-x^2} + C$   
 or  $-\frac{1}{3}\sqrt{a^2-x^2}(2a^2+x^2) + C$     21  $\frac{1}{2}\sqrt{x^4-1} - \frac{1}{2} \operatorname{arccsc}(x^2) + C$   
 23  $\sqrt{a^2+x^2} + \frac{a^2}{\sqrt{a^2+x^2}} + C$     25  $\pi/2$     27  $1/9$     29  $\infty$   
 31  $(\frac{1}{2}x^2 - \frac{1}{4}) \arcsin x + \frac{1}{4}x\sqrt{1-x^2} + C$     33  $\frac{1}{3}x^3 \arcsin x + \frac{1}{3}\sqrt{1-x^2} - \frac{1}{9}(1-x^2)^{3/2} + C$   
 35  $-\frac{\arcsin x}{2x^2} - \frac{\sqrt{1-x^2}}{2x} + C$     37  $\frac{2\pi^2}{3\sqrt{3}} + \frac{\pi}{2}$

## Section 7.7

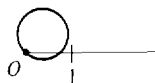


- 3  $r = \frac{2}{\sin \theta - 5 \cos \theta}$     5  $r = 2 \sec \theta$     7  $r \sin \theta = r^2 \cos^2 \theta + 1$   
 9  $r \sin \theta = 3r^2 \cos^2 \theta - 2r \cos \theta$  or  $r = \frac{\sin \theta + 2 \cos \theta}{3 \cos^2 \theta}$     11  $r = \sin \theta$

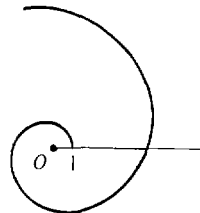
13



15

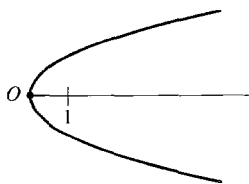


17





19

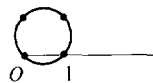


21  $x = \sin(3\theta) \cos \theta, y = \sin(3\theta) \sin \theta$     23  $x = \theta^2 \cos \theta, y = \theta^2 \sin \theta$

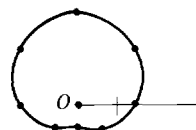
Section 7.8

1  $\theta$     3  $-\cot \theta$     5  $-\frac{1 + \cot \theta}{\sin \theta}$

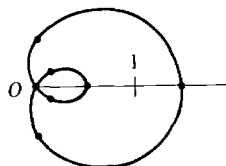
7	$\theta$	$r$	$dr/d\theta$	$\tan \psi$	$ r $
	0	1	1	1	incr.
	$\pi/4$	$\sqrt{2}$	0		max.
	$\pi/2$	1	-1	-1	decr.
	$3\pi/4$	0	$-\sqrt{2}$		crosses O
	$\pi$	-1	-1	1	incr.
	$5\pi/4$	$-\sqrt{2}$	0		max.
	$3\pi/2$	-1	1	-1	decr.
	$7\pi/4$	0	$\sqrt{2}$		crosses O
	$2\pi$	1	1	1	incr.



9	$\theta$	$r$	$dr/d\theta$	$\tan \psi$	$ r $
	0	1.5	1	1.5	incr.
	$\pi/4$	2.2	0.7	3.1	incr.
	$\pi/2$	2.5	0		max.
	$3\pi/4$	2.2	-0.7	-3.1	decr.
	$\pi$	1.5	-1	-1.5	decr.
	$5\pi/4$	0.8	-0.7	-1.1	decr.
	$3\pi/2$	0.5	0		min.
	$7\pi/4$	0.8	0.7	1.1	incr.
	$2\pi$	1.5	1	1.5	incr.

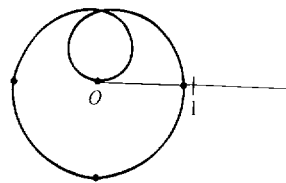


11	$\theta$	$r$	$dr/d\theta$	$\tan \psi$	$ r $
	0	1.5	0		max.
	$\pi/2$	0.5	-1	-0.5	decr.
	$2\pi/3$	0	-0.9		crosses O
	$3\pi/4$	-0.2	-0.7	0.3	incr.
	$\pi$	-0.5	0		max.
	$5\pi/4$	-0.2	0.7	-0.3	decr.
	$4\pi/3$	0	0.9		crosses O
	$3\pi/2$	0.5	1	0.5	incr.
	$2\pi$	1.5	0		max.



13

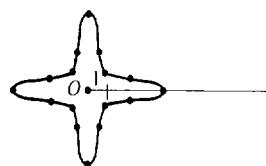
$\theta$	$r$	$dr/d\theta$	$\tan \psi$	$ r $
0	0	$\frac{1}{3}$		crosses $O$
$\pi$	0.9	$\frac{1}{6}$	5	incr.
$1.5\pi$	1	0		max.
$2\pi$	0.9	$-\frac{1}{6}$	-5	decr.
$3\pi$	0	$-\frac{1}{3}$		crosses $O$
$4\pi$	-0.9	$-\frac{1}{6}$	5	incr.
$4.5\pi$	-1	0		max.
$5\pi$	-0.9	$\frac{1}{6}$	-5	decr.
$6\pi$	0	$\frac{1}{3}$		crosses $O$



15

$\theta$	$r$	$dr/d\theta$	$\tan \psi$	$ r $
0	4	0		max.
$\pi/8$	2.5	-6	-0.4	decr.
$2\pi/8$	1	0		min.
$3\pi/8$	2.5	6	0.4	incr.
$4\pi/8$	4	0		max.
$5\pi/8$	2.5	-6	-0.4	decr.
$6\pi/8$	1	0		min.
$7\pi/8$	2.5	6	0.4	incr.
$\pi$	4	0		max.

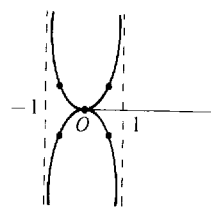
repeats



17

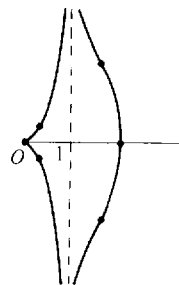
$\theta$	$r$	$dr/d\theta$	$\tan \psi$	$ r $
0	0	1		crosses $O$
$\pi/4$	1	2	$\frac{1}{2}$	incr.
$\lim_{\theta \rightarrow \pi/2^-}$	$\infty$			$x \rightarrow 1$
$\lim_{\theta \rightarrow \pi/2^+}$	$-\infty$			$x \rightarrow 1$
$3\pi/4$	-1	2	$-\frac{1}{2}$	decr.
$\pi$	0	1		crosses $O$

repeats, with  $x \rightarrow -1$  as  $\theta \rightarrow 3\pi/2$

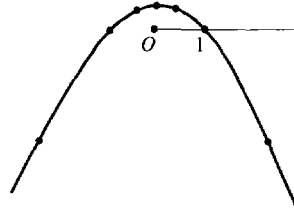


19

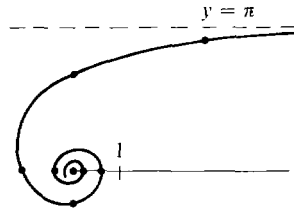
$\theta$	$r$	$dr/d\theta$	$\tan \psi$	$ r $
0	2	0		min.
$\pi/4$	2.4	2.4	1	incr.
$\lim_{\theta \rightarrow \pi/2^-}$	$\infty$			$x \rightarrow 1$
$\lim_{\theta \rightarrow \pi/2^+}$	$-\infty$			$x \rightarrow 1$
$3\pi/4$	-0.4	0.4	-1	decr.
$\pi$	0	0		cusp at $O$
$5\pi/4$	-0.4	-0.4	1	incr.
$\lim_{\theta \rightarrow 3\pi/2^-}$	$-\infty$			$x \rightarrow 1$
$\lim_{\theta \rightarrow 3\pi/2^+}$	$\infty$			$x \rightarrow 1$
$7\pi/4$	2.4	-2.4	-1	decr.
$2\pi$	2	0		min.



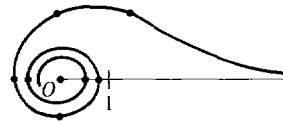
21	$\theta$	$r$	$dr/d\theta$	$\tan \psi$	$ r $
	0	1	-1	-1	decr.
	$\pi/4$	0.6	-0.3	-2	decr.
	$\pi/2$	0.5	0		min.
	$3\pi/4$	0.6	0.3	2	incr.
	$\pi$	1	1	1	incr.
	$5\pi/4$	3.4	8	0.4	incr.
	$\lim_{\theta \rightarrow 3\pi/2^-}$	$\infty$			$x \rightarrow -\infty$
	$\lim_{\theta \rightarrow 3\pi/2^+}$	$\infty$			$x \rightarrow \infty$
	$7\pi/4$	3.4	-8	-0.4	decr.
	$2\pi$	1	-1	-1	decr.



23	$\theta$	$r$	$dr/d\theta$	$\tan \psi$	$ r $
	$\lim_{\theta \rightarrow 0^+}$	$\infty$			$y \rightarrow \pi$
	$\pi/4$	4	$-16/\pi$	$-\pi/4$	decr.
	$\pi/2$	2	$-4/\pi$	$-\pi/2$	decr.
	$\pi$	1	$-1/\pi$	$-\pi$	decr.
	$3\pi/2$	$\frac{2}{3}$	$-\frac{4}{3}\pi$	$-3\pi/2$	decr.
	$2\pi$	$\frac{1}{2}$	$-\frac{1}{2}\pi$	$-2\pi$	decr.
	$3\pi$	$\frac{1}{3}$	$-\frac{1}{3}\pi$	$-3\pi$	decr.
	$4\pi$	$\frac{1}{4}$	$-\frac{1}{4}\pi$	$-4\pi$	decr.



25	$\theta$	$r$	$dr/d\theta$	$\tan \psi$	$ r $
	$\lim_{\theta \rightarrow 0^+}$	$\infty$			$y \rightarrow 0$
	$\pi/4$	2	$-4/\pi$	$-\pi/2$	decr.
	$\pi/2$	$\sqrt{2}$	$-\sqrt{2}/\pi$	$-\pi$	decr.
	$\pi$	1	$-\frac{1}{2}\pi$	$-2\pi$	decr.
	$3\pi/2$	$\sqrt{\frac{2}{3}}$	-	-	decr.
	$2\pi$	$1/\sqrt{2}$	-	-	decr.
	$3\pi$	$1/\sqrt{3}$	-	-	decr.
	$4\pi$	$\frac{1}{2}$	-	-	decr.



27  $x$ : max of  $\sqrt{32/27}$  at  $\theta = \arcsin(1/\sqrt{3})$ ,  $2\pi - \arcsin(1/\sqrt{3})$ ; min of  $-\sqrt{32/27}$  at  $\theta = \pi + \arcsin(1/\sqrt{3})$ ,  $\pi - \arcsin(1/\sqrt{3})$

$y$ : max of 2 at  $\theta = \pi/2$ ; min of -2 at  $\theta = 3\pi/2$

29  $x$ : max of  $\frac{5}{2}$  at  $\theta = 0$ ; min of  $-\frac{9}{16}$  at  $\theta = \arccos(-\frac{3}{4})$ ,  $2\pi - \arccos(-\frac{3}{4})$

$y$ : max of  $\sin \theta_0(\frac{3}{2} + \cos \theta_0)$  at  $\theta = \theta_0 = \arccos(\frac{\sqrt{41}-3}{8})$ ; min of  $-\sin \theta_0(\frac{3}{2} + \cos \theta_0)$  at  $\theta = 2\pi - \arccos(\frac{\sqrt{41}-3}{8})$

31  $(\frac{1}{2}, \frac{\pi}{12})$ ,  $(\frac{1}{2}, \frac{5\pi}{12})$ ,  $(\frac{1}{2}, \frac{13\pi}{12})$ ,  $(\frac{1}{2}, \frac{17\pi}{12})$ ,  $(\frac{1}{2}, \frac{7\pi}{12})$ ,  $(\frac{1}{2}, \frac{11\pi}{12})$ ,  $(\frac{1}{2}, \frac{19\pi}{12})$ ,  $(\frac{1}{2}, \frac{23\pi}{12})$

Section 7.9

1  $\pi a^2$     3 1    5  $2 - \frac{\pi}{2}$     7  $\frac{3\pi}{16}$     9  $\frac{\pi}{4} - \frac{3\sqrt{3}}{8}$     11  $\frac{\pi}{12} + \frac{\sqrt{3}}{16}$     13  $\frac{3}{2} - \frac{\pi}{4}$

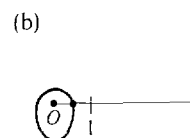
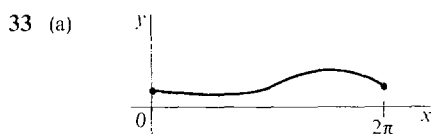
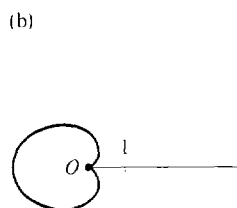
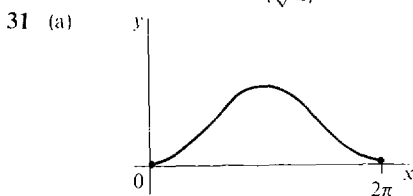
15  $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$     17  $\pi^2$     19  $\frac{7\pi}{12} - \sqrt{3}$     21  $\frac{1}{2} \int_a^b (g(\theta))^2 - (f(\theta))^2 d\theta$

## Section 7.10

- 1  $14\pi$     3  $2$     5  $\frac{2048 - 493\sqrt{17}}{15} \sim 1.02$     7  $4$     9  $4$
- 11  $\int_0^{2\pi} \sqrt{\sin^2(2\theta) + 4\cos^2(2\theta)} d\theta$     13  $\int_0^b \sqrt{\theta^2 + 1} d\theta$     17  $\pi\sqrt{a^2 + b^2} \left( a + \frac{b\pi}{2} \right)$
- 19  $\pi\sqrt{2}$     21  $\frac{\pi}{5}(8 - \sqrt{2})$

## Extra Problems for Chapter 7

- 1  $1 + \cos x$     3  $\frac{\cos \theta}{2\sqrt{\theta}} - \sqrt{\theta} \sin \theta$     5  $4/3$     7  $-\sin(\cos \theta) + C$     9  $8$
- 11  $-600/29$  radians/hr    13  $-\frac{1}{2\sqrt{x}\sqrt{1-x}}$     15  $\frac{1}{1+t^2} - 1$  or  $-\frac{t^2}{1+t^2}$     17  $1$
- 19  $\operatorname{arcsec}(x-1) + C$     21  $\frac{1}{3}(x^2+1)^{3/2} - (x^2+1)^{1/2} + C = x^2(x^2+1)^{1/2} - \frac{2}{3}(x^2+1)^{3/2} + C = (x^2+1)^{1/2}(\frac{1}{3}x^2 - \frac{2}{3}) + C$     23  $2\sin 1 + 2\cos 1 - 2$
- 25  $\pi\left(1 - \frac{\pi}{4}\right)$     27  $\frac{\sec^9 \theta}{9} - \frac{2\sec^7 \theta}{7} + \frac{\sec^5 \theta}{5} + C$
- 29  $-\frac{\sqrt{2-x^2}}{x} - \arcsin\left(\frac{x}{\sqrt{2}}\right) + C$



- 35  $19\pi/8$     37  $(3\pi/2) - 4$     39  $4\sqrt{2}\pi/5$

## Section 8.1

- 9  $0$     11  $\infty$     13  $0$     15  $\infty$     17  $27$     19  $\infty$     21  $1/6$     23  $1/(2\pi)$

## Section 8.2

- 1  $x$     3  $-x^2$     5  $x/y^2$     7  $\frac{2}{3}\log_b x$     9  $\frac{1}{2}(\log_b x + \log_b y)$     11  $2$     13  $1/2$
- 15  $x = \log_5 3$     17  $x = -\frac{2}{3}$     19  $x = 5^{1/3}$     21  $x = 2, x = 3$
- 23  $x = 2^{1/(1-\log_3 2)} = 3^{1/(\log_3 3 - 1)}$     25  $-\infty$     27  $-\infty$     29  $-\infty$

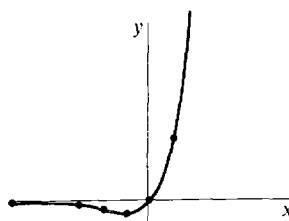
## Section 8.3

- 1  $3e^{3x+4}$     3  $-4^{-x} \ln 4$     5  $e^t \cos(e^t)$     7  $2^{t^2+1} t \ln 2$     9  $e^t e^{(e^t)}$
- 11  $\frac{\ln 3}{2\sqrt{x}} 3^{\sqrt{x}}$     13  $-\frac{e^{x+y}}{e^{x+y} + \sin y}$     15  $\frac{1}{2\sqrt{e^t(t^{-1} - t^{-2})}}$  or  $\frac{t^2}{2\sqrt{e^t(t-1)}}$     17  $\infty$
- 19  $1$     21  $-\infty$     23  $e^e$     25  $e^e$

$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	0	0			
-2	1/4	$(\ln 2)/4$	+	incr.	∪
-1	1/2	$(\ln 2)/2$	+	incr.	∪
0	1	$\ln 2$	+	incr.	∪
1	2	$2 \ln 2$	+	incr.	∪
2	4	$4 \ln 2$	+	incr.	∪
$\lim_{x \rightarrow \infty}$	$\infty$	$\infty$			



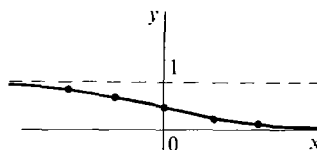
$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	0	0			
-3	$-3e^{-3}$	$-2e^{-3}$	-	decr.	∩
-2	$-2e^{-2}$	$-e^{-2}$	0	decr.	infl.
-1	$-e^{-1}$	0	+	min.	∪
0	0	1	+	incr.	∪
1	$e$	$2e$	+	incr.	∪
$\lim_{x \rightarrow \infty}$	$\infty$	$\infty$			



$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	0	0			
-2	$e^{-8}$	$12e^{-8}$	+	incr.	∪
-1	$e^{-1}$	$3e^{-1}$	+	incr.	∪
$-(\frac{2}{3})^{1/3}$	$e^{-2/3}$	+	0	incr.	infl.
$-\frac{1}{2}$	$e^{-1/8}$	$\frac{3}{4}e^{-1/8}$	-	incr.	∩
0	1	0	0	horiz.	infl.
1	$e$	$3e$	+	incr.	∪
$\lim_{x \rightarrow \infty}$	$\infty$	$\infty$			



$x$	$f(x)$	$f'(x)$	$f''(x)$		
$\lim_{x \rightarrow -\infty}$	1	0			
-2	0.88	-0.1	-	decr.	∩
-1	0.73	-0.2	-	decr.	∩
0	0.5	-0.25	0	decr.	infl.
1	0.27	-0.2	+	decr.	∪
2	0.12	-0.1	+	decr.	∪
$\lim_{x \rightarrow \infty}$	0	0			



35  $\frac{1}{2}e^{2x} + C$     37  $-\frac{1}{2}e^{-x^2} + C$     39  $\frac{1}{3}(1 + e^{2x})^{3/2} + C$     41  $xe^x - e^x + C$

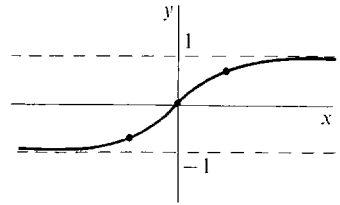
43  $\frac{1}{2}e^x(\sin x - \cos x) + C$     45  $\frac{e^{10} - 1}{5}$     47  $\infty$

49  $\frac{1}{r^2}$  if  $r > 0$ ,  $\infty$  if  $r \leq 0$     51 (a)  $\frac{\pi}{2}(e^2 - 1)$  (b)  $2\pi$     53  $\sqrt{2}(e^{2\pi} - 1)$

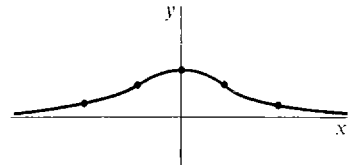
## Section 8.4

1  $3 \cosh(3x)$     3  $-\tanh x \operatorname{sech} x$     5 1    7 0

9	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow -\infty}$	-1	0			
	-1	-0.76	0.4	+	incr.	$\cup$
	0	0	1	0	incr.	infl.
	1	0.76	0.4	-	incr.	$\cap$
	$\lim_{x \rightarrow \infty}$	1	0			



11	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow -\infty}$	0	0			
	-2	0.27	0.26	+	incr.	$\cup$
	-0.88	0.71	0.5	0	incr.	infl.
	0	1	0	-	max.	$\cap$
	0.88	0.71	-0.5	0	decr.	infl.
	2	0.27	-0.26	+	decr.	$\cup$
	$\lim_{x \rightarrow \infty}$	0	0			



13  $\frac{\sinh^2 x}{2} + C$  or  $\frac{\cosh^2 x}{2} + C$     15  $x \cosh x - \sinh x + C$

17  $\frac{1}{2}x \sinh x \cosh x + \frac{1}{4}x^2 - \frac{1}{4}\sinh^2 x + C$  or  $\frac{1}{4}x \sinh(2x) - \frac{1}{8}\cosh(2x) + \frac{1}{4}x^2 + C$     19  $\infty$

23 (a)  $\frac{\pi}{8}(4 + e^2 - e^{-2})$  (b)  $2\pi(1 - e^{-1})$     25  $50,000(e^2 - 1) \sim 320,000$  dollars

27 (a)  $10^6(90e^{0.2} - 110) \sim -74,000$  dollars (b)  $10^6(90e^{0.3} - 120) \sim 1,490,000$  dollars

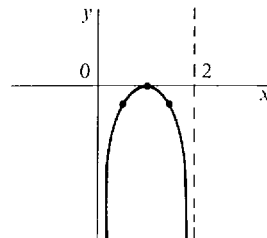
## Section 8.5

1  $(3/x)(\ln x)^2$     3  $-\tan x$     5  $\ln t$     7  $t^{-2}(1 - \ln t)$     9  $\frac{1}{x \ln 2}$

11  $\frac{1}{y} + \frac{3}{2(3y+1)}$     13  $y - \frac{y}{x}$     15  $\frac{1}{x+y-1}$     17 0    19 0    21  $\ln a$

23 1    25 1

27	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow 0^+}$	$-\infty$	$\infty$			
	$\frac{1}{2}$	-0.3	$\frac{4}{3}$	-	incr.	$\cap$
	1	0	0	-	max.	$\cap$
	$\frac{3}{2}$	0.3	$-\frac{4}{3}$	-	decr.	$\cap$
	$\lim_{x \rightarrow 2^-}$	$-\infty$	$-\infty$			



29  $\frac{1}{2} \ln |2x + 3| + C$     31  $2x + 2 \ln |x - 1| + C$     33  $\frac{1}{2}(\ln x)^2 + C$     35  $\ln |\ln t| + C$

37  $\ln x(\ln(\ln x) - 1) + C$     39  $\frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + C$

41  $x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x - 6x + C$     43  $\sin(\ln x) + C$     45  $10 - \ln(11)$

47 1    49  $-\infty$     51  $\infty$     53 (a)  $\pi(e-2)$  (b)  $\frac{\pi}{2}(e^2+1)$

55  $\sqrt{e^2+1} - \sqrt{2} + 1 + \ln\left(\frac{1+\sqrt{2}}{1+\sqrt{e^2+1}}\right)$

## Section 8.6

1  $y = -\frac{2}{x^2+C}$     3  $y = \pm\sqrt{\frac{2}{3}x^3+C}$     5  $y = -\ln\left(\frac{C-x^2}{2}\right)$

7  $y = \ln(e^x+C)$     9  $y = \frac{1}{\pm\sqrt{2\cos x+C}}$     11  $y = \frac{x^3}{3} + \frac{x^2}{2} + Ax + B$

13  $y = Ax + B$     15  $y = A \sinh(\sqrt{3}x) + B \cosh(\sqrt{3}x)$  or  $y = Ae^{\sqrt{3}x} + Be^{-\sqrt{3}x}$

17  $y = 10^7 e^{(0.01)t}$     19  $\frac{6 \ln 10}{\ln 2} \text{ min} \sim 19.93 \text{ min}$     21  $10^5 \times (1.15)^4 \sim 174,900$

23  $t = \frac{\ln(0.5)}{\ln(0.9)} \sim 6.576 \text{ years}$     25  $y = e^t + e^{-t}$     27  $y = \cos t - 2 \sin t$

29  $y = 10^{6.5} \sqrt{t}$     31  $y = \frac{10^8}{1+9e^{-t}}$     33 (b)  $y_0 = L/2$

35 (a)  $y = 10^7 - 6 \times 10^{62-t}$  (b)  $y = 10^7 + 3 \times 10^{63-t}$

37  $y^2 - x^2 = C$  or  $y = \pm\sqrt{x^2+C}$

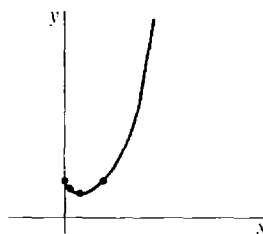
## Section 8.7

1  $\left(\frac{3x-2}{4x+3}\right) \left[\frac{3}{3x-2} - \frac{4}{4x+3}\right]$

3  $\frac{(x^2+1)\sqrt{3x+4}}{(2x-3)\sqrt{x^2-4}} \left[\frac{2x}{x^2+1} + \frac{3}{2(3x+4)} - \frac{2}{2x-3} - \frac{x}{x^2-4}\right]$

5  $(x-1)^{x^2+1} \left[2x \ln(x-1) + \frac{x^2+1}{x-1}\right]$     7  $e^{(e^x)}e^x$     9  $\sqrt[t]{t} \left[\frac{1}{t^2} - \frac{1}{t^2} \ln t\right]$

11	$x$	$f(x)$	$f'(x)$	$f''(x)$		
	$\lim_{x \rightarrow 0^+}$	1	$-\infty$			
	1/10	$10^{-0.1}$	-	+	decr.	∪
	1/e	$e^{-1/e}$	0	+	min.	∪
	1	1	1	+	incr.	∪
	$\lim_{x \rightarrow \infty}$	$\infty$	$\infty$			



15  $\ln|\sin \theta| + C$     17  $\frac{1}{3} \ln|\sec(3\theta)| + C$

19  $\frac{1}{4} \sec^4 \theta \sin \theta + \frac{3}{8} \sec^2 \theta \sin \theta + \frac{3}{8} \ln|\sec \theta + \tan \theta| + C$     21  $\ln|\tan x| + C$

23  $\ln|\sec x + \tan x| - \sin x + C$     25  $\frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2} \ln|x + \sqrt{x^2-1}| + C$

27  $\frac{1}{2} \ln \left| \frac{x}{\sqrt{4-x^2}+2} \right| + C$  or  $\frac{1}{4} \ln \left| \frac{2-\sqrt{4-x^2}}{2+\sqrt{4-x^2}} \right| + C$     29  $\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + C$

31  $\sqrt{x^2-1} - \operatorname{arcsec} x + C$  or  $\sqrt{x^2-1} - \arctan \sqrt{x^2-1} + C$

33  $\frac{x}{4}(x^2+1)^{3/2} - \frac{x}{8}(x^2+1)^{1/2} - \frac{1}{8} \ln(x + \sqrt{x^2+1}) + C$  or

$x(x^2+1)^{1/2} \left(\frac{1}{4}x^2 + \frac{1}{8}\right) - \frac{1}{8} \ln(x + \sqrt{x^2+1}) + C$

- 35  $-\frac{\arcsin x}{x} - \ln \left| \frac{1 + \sqrt{1-x^2}}{x} \right| + C$  or  $-\frac{\arcsin x}{x} + \ln \left| \frac{1 - \sqrt{1-x^2}}{x} \right| + C$   
 37  $x \operatorname{arccsc} x + \ln(x + \sqrt{x^2-1}) + C, x \geq 1; x \operatorname{arccsc} x - \ln|x + \sqrt{x^2-1}| + C, x \leq -1$   
 39  $\sqrt{5} + \frac{1}{2} \ln(2 + \sqrt{5}) \sim 2.96$     41  $\frac{1}{2}a\sqrt{1+a^2} + \frac{1}{2} \ln(a + \sqrt{1+a^2})$

## Section 8.8

- 1  $\frac{1}{2} \ln|2x-7| + C$     3  $\frac{1}{2} \ln|x-4| - \frac{1}{2} \ln|x| + C$     5  $-\frac{1}{5} \ln|x-1| + \frac{1}{5} \ln|x+4| + C$   
 7  $\frac{x^2}{2} - 3x + \frac{1}{4} \ln|x| + \frac{5}{4} \ln|x+4| + C$     9  $-\frac{1}{2(x+1)^2} + C$   
 11  $\ln|x-1| - \frac{1}{x-1} - \frac{1}{2(x-1)^2} + C$     13  $-\ln|x| + \frac{1}{2} \ln|x+1| + \frac{1}{2} \ln|x-1| + C$   
 15  $x + \ln|x| - \frac{1}{x} + C$     17  $x - 4 \arctan(x/4) + C$     19  $\frac{5}{4} \ln|x-3| - \frac{1}{4} \ln|x+1| + C$   
 21  $\frac{1}{3}x^3 - x + \arctan x + C$     23  $\arctan x - (1/\sqrt{2}) \arctan(x/\sqrt{2}) + C$   
 25  $-x^{-1} - \arctan x + C$     27  $\frac{1}{3} \ln|x+1| - \frac{1}{6} \ln|x^2-x+1| + \frac{1}{\sqrt{3}} \arctan\left(\frac{2x-1}{\sqrt{3}}\right) + C$   
 29  $\frac{x^2}{2} - \frac{3}{4} \ln|x+1| + \frac{5}{4} \ln|x-1| - \frac{1}{4} \ln|x^2+1| + \arctan x + C$   
 31  $\ln|x| - \frac{1}{2} \ln|x^2+1| - \frac{\arctan x}{x} + C$

## Section 8.9

- 1  $-3 \cos x + 4 \sin x + C$     3  $\frac{3}{4}(x^2-1)^{2/3} + C$     5  $\frac{1}{3}[(x+2)^{3/2} + x^{3/2}] + C$   
 7  $2(1 + \sqrt{x}) - 2 \ln(1 + \sqrt{x}) + C$  or  $2\sqrt{x} - 2 \ln(1 + \sqrt{x}) + C$   
 9  $-(\sqrt{4x+1}/x) + C$     11  $-2 \cos(\sqrt{x}) + C$   
 13  $\frac{2}{3}(x-3)^{7/2} + \frac{1}{5}(x-3)^{5/2} + 6(x-3)^{3/2} + C$   
 15  $\frac{1}{3}[(3x+4) \ln(3x+4) - (3x+4)] + C$  or  $\frac{1}{3}(3x+4) \ln(3x+4) - x + C$   
 17  $x \tan x - \ln|\sec x| - (x^2/2) + C$     19  $-\frac{1}{6} \cos(3x^2+1) + C$   
 21  $2x \ln|x| - 3x + (x+1) \ln(x+1) + C$     23  $\cos \theta - \cos \theta \ln(\cos \theta) + C$   
 25  $\ln(2 - \cos \theta) + C$     27  $\frac{1}{4}(e^x+1)^4 + C$     29  $2 \sin(\sqrt{x}) - 2\sqrt{x} \cos(\sqrt{x}) + C$   
 31  $\ln(1 + \cosh x) + C$     33  $-\frac{\ln x}{1+x} + \ln\left(\frac{x}{1+x}\right) + C$   
 35  $\frac{4}{5}(1 - \cos x)^{5/2} - \frac{2}{7}(1 - \cos x)^{7/2} + C$     37  $\frac{1}{\sqrt{2}} \ln \left| \frac{x}{\sqrt{2} + \sqrt{2+x^2}} \right| + C$   
 39  $4 \arcsin\left(\frac{\sqrt{x}}{2}\right) + \sqrt{x}\sqrt{4-x} - \frac{1}{2}\sqrt{x}(4-x)^{3/2} + C$  or  
 $\frac{1}{2}(x-2)\sqrt{x}\sqrt{4-x} + 2 \arcsin\left(\frac{x-2}{2}\right) + C$   
 41  $\frac{1}{5}[(5x-2) \arcsin(5x-2) + \sqrt{1-(5x-2)^2}] + C$     43  $\frac{1}{2}e^x(\sin x + \cos x) + C$   
 45  $(x^3/3) - x + 2 \arctan x + C$     47  $\frac{1}{2}x^2 \operatorname{arcsec}(x^2) - \frac{1}{2} \ln(x^2 + \sqrt{x^4-1}) + C$   
 49  $2x \ln x + \frac{4x-1}{8} \ln(4x-1) - \frac{5}{2}x + C$     51  $2 \ln|2x + \sqrt{4x^2-1}| - \frac{\sqrt{4x^2-1}}{x} + C$   
 53  $(x+1) \arctan(\sqrt{x}) - \sqrt{x} + C$     55  $\frac{1}{8} \ln|\sec(4x^2+7) + \tan(4x^2+7)| + C$   
 57  $-\frac{1}{243}\sqrt{1-9x^2}(2+9x^2) + C$     59  $(\sqrt{x^2-3}/3x) + C$   
 61  $3x^{2/3} \sin(\sqrt[3]{x}) + 6(\sqrt[3]{x}) \cos(\sqrt[3]{x}) - 6 \sin(\sqrt[3]{x}) + C$   
 63  $\frac{1}{3}x^3(1-x^2)^{-3/2} + x(1-x^2)^{-1/2} + C$     65  $\frac{x}{5}(\cos^2(\ln x) + \sin(2 \ln x) + 2) + C$



## Extra Problems for Chapter 8

- 1  $-\infty$     3  $-e^{\cos \theta} \sin \theta$     5  $-3 \operatorname{csch}^3 x \coth x$     7  $-\frac{1}{\ln 3} \cos(3^x) + C$   
 9  $\frac{1}{2} e^x \sqrt{1 - e^{2x}} + \frac{1}{2} \arcsin(e^x) + C$  or  $\frac{1}{2} e^x \sqrt{1 - e^{2x}} - \frac{1}{2} \arccos(e^x) + C$   
 11  $\frac{1}{4} e^{2x} - \frac{1}{2} x + C$     13  $\frac{8x}{x^2 - 1}$     15  $\frac{3}{3x + 2} + \frac{5}{5x - 4} - \frac{2}{2x - 1} - \frac{2x}{x^2 + 1}$     17  $e^2$   
 19  $(1/a) \ln |x| - (b/a) \ln |a + bx| + C$     21  $\infty$     23  $y = \pm \sqrt{ax^2 + C}$   
 25  $P = CV^{-1/k}$     27 air temp. =  $60^\circ$ ,  $y = 60 + 80 \cdot 2^{-t/10}$   
 29  $(4t + 1)(t - 3)^{2t+1} \left\{ \ln [(t - 3)^2(4t + 1)] + \frac{4t}{4t + 1} + \frac{2t + 1}{t - 3} \right\}$     31  $\ln |\cosh x| + C$   
 33  $\frac{2}{3}(x + 1)^{5/2} + C$     35  $\pi(2\sqrt{2} + \ln|\sqrt{2} + 1| - \ln|\sqrt{2} - 1|)$  or  $2\pi(\sqrt{2} + \ln|\sqrt{2} + 1|)$   
 37  $e^{0.03} \sim 1.03$ , error  $\leq 0.0005$     39  $\ln 6 \sim \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{12} = \frac{28}{15}$ , error  $\leq \frac{1}{2}$   
 41  $\bar{x} = \frac{2 \ln 2 - \frac{3}{4}}{2 \ln 2 - 1}$ ,  $\bar{y} = \frac{(\ln 2 - 1)^2}{2 \ln 2 - 1}$   
 43  $\pi[e\sqrt{1 + e^2} - \sqrt{2} + \ln(e + \sqrt{1 + e^2}) - \ln(1 + \sqrt{2})]$

## Section 9.1

- 1  $2^{-n}$     3  $(-1)^n n$     5  $2 - (1/2^{n-1})$     7  $2^{2^{(n-1)}}$     9 diverges    11 1  
 13 0    15 diverges    17 diverges    19  $1/e$     21 0    23 -1    25 diverges  
 27 diverges    29 0

## Section 9.2

- 1  $\frac{3}{2}(1 - (1/3)^n)$ , converges to  $\frac{3}{2}$     3  $4(1 - (3/4)^n)$ , converges to 4  
 5  $1 - \frac{1}{(n+1)!}$ , converges to 1    7  $1 - \frac{1}{n+1}$ , converges to 1  
 9  $n$  even:  $-\frac{n}{2}$ ,  $n$  odd:  $\frac{n}{2} + \frac{1}{2}$ , diverges    11  $1 - (n+1)^{-2}$ , converges to 1  
 13  $\frac{1}{2} \left( 1 - \frac{1}{2n+1} \right)$ , converges to  $\frac{1}{2}$     15 diverges because  $\lim_{n \rightarrow \infty} a_n = \frac{1}{2}$   
 17 diverges because for infinite  $H$ ,  $S_{2H} \not\approx S_{2 \cdot 2H}$   
 19 diverges because for infinite  $H$ ,  $S_{2H} \not\approx S_H$     21  $A: 2/3, B: 1/3$   
 23  $A: 9/19, B: 6/19; C: 4/19$

## Section 9.3

- 1  $1/42$     3  $11/4$     5  $5/48$     7  $216/5$     9  $80/9$     11  $5\frac{4}{9}$   
 13  $492.315 + (41/999,000)$     15  $1/7$

## Section 9.4

- 1 diverges    3 converges    5 converges    7 converges    9 diverges  
 11 diverges    13 converges    15 diverges    17 converges    19 converges  
 21 diverges    23 converges    25 converges    27 diverges    29 diverges  
 31 converges    33 diverges    35 converges    37 converges

## Section 9.5

- 1 diverges    3 diverges    5 converges    7 diverges    9 diverges  
 11 diverges    13 converges    15 converges    17 converges    19 diverges  
 21 0.90    23 0.37

## Section 9.6

- 1 diverges    3 diverges    5 absolutely converges    7 diverges    9 diverges  
 11 diverges    13 conditionally converges    15 absolutely converges  
 17 conditionally converges    19 diverges    21 diverges    23 no information  
 25 converges    27 diverges    29 converges    31 converges    33 no information  
 35 converges    37 converges    39 converges    41 no information    43 converges

## Section 9.7

- 1 1    3 0    5  $e$     7  $4/e^2$     9 1    11 1    13 0    15  $1/3$     17  $\infty$   
 19  $\infty$     21 0    23  $\sqrt[3]{5}$     25 1    27  $(-1, 1)$     29  $(-1, 1]$     31  $[-1/2, 1/2)$   
 33  $(-\infty, \infty)$     35  $(-3, -1]$     37  $(-\infty, \infty)$     39  $(-1, 1)$     41  $(-5/4, 5/4)$   
 43  $(-\sqrt{5}, \sqrt{5})$     45  $(-\infty, \infty)$

## Section 9.8

- 1  $f'(x) = \sum_{n=1}^{\infty} n 10^n x^{n-1}$ ,  $\int_0^x f(t) dt = \sum_{n=0}^{\infty} \frac{10^n}{n+1} x^{n+1}$   
 3  $f'(x) = \sum_{n=1}^{\infty} n^{-2} x^{n-1}$ ,  $\int_0^x f(t) dt = \sum_{n=1}^{\infty} \frac{n^{-3}}{n+1} x^{n+1}$   
 5  $f'(x) = \sum_{n=1}^{\infty} (n+1)x^{n-1}$ ,  $\int_0^x f(t) dt = \sum_{n=1}^{\infty} \frac{1}{n} x^{n+1}$   
 7  $f'(x) = \sum_{n=1}^{\infty} \frac{n!}{n^{n-1}} x^{n-1}$ ,  $\int_0^x f(t) dt = \sum_{n=1}^{\infty} \frac{n!}{n^n(n+1)} x^{n+1}$   
 9  $f'(x) = \sum_{n=1}^{\infty} 2n x^{2n-1}$ ,  $\int_0^x f(t) dt = \sum_{n=0}^{\infty} \frac{1}{2n+1} x^{2n+1}$     11  $\sum_{n=0}^{\infty} (-1)^n 3^n x^n$ ,  $r = \frac{1}{3}$   
 13  $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n+1} x^{4n+2}}{2n+1}$ ,  $r = \frac{1}{2}$     15  $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} x^{n+2}}{n+1}$ ,  $r = \frac{1}{2}$   
 17  $\sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^n}{n!}$ ,  $r = \infty$     19  $\sum_{n=0}^{\infty} \frac{3^{2n+1} x^{2n+1}}{(2n+1)!}$ ,  $r = \infty$   
 21  $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} x^{2n+3}}{(n+1)(2n+3)}$ ,  $r = \frac{1}{\sqrt{2}}$     23  $\sum_{n=0}^{\infty} \frac{x^{3n+1}}{n!(3n+1)}$ ,  $r = \infty$   
 25  $\sum_{n=0}^{\infty} \frac{x^{n+3}}{(n+1)(n+3)}$ ,  $r = 1$     27  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)^2}$ ,  $r = 1$   
 29  $\sum_{n=1}^{\infty} (-1)^{n+1} 2n x^{2n-1}$ ,  $r = 1$     31  $\sum_{n=0}^{\infty} (-1)^n 2 \cdot x^{4n+1}$ ,  $r = 1$   
 33  $\sum_{n=0}^{\infty} \frac{(-1)^n (1+2^{2n+1}) x^{2n+1}}{2n+1}$ ,  $r = \frac{1}{2}$

## Section 9.9

- 1 0.18232    3 0.7788008    5 0.4854019    7 0.3293740    9 1.003009  
 11 1.098614    13  $f(x) = x + x^2 + \cdots + x^{n+1} + E$ ,  $|E| \leq 2|x|^{n+2}$ ,  $f(\frac{1}{2}) = 1$   
 15  $f(x) = 1 + x^2 + \cdots + x^{2n} + E$ ,  $|E| \leq \frac{4}{3}x^{2n+2}$ ,  $f(\frac{1}{2}) = \frac{4}{3}$   
 17  $f(x) = -x - (x^2/2^2) - \cdots - (x^n/n^2) + E$ ,  $|E| \leq 2|x|^{n+1}/(n+1)^2$ ,  $f(\frac{1}{2}) \sim 0.58$

## Section 9.10

- 1  $f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \frac{(-1)^{n+1} (\cos t) x^{2n+2}}{(2n+2)!}$ ,  $f\left(\frac{1}{2}\right) \sim 0.8776$

$$3 \quad f(x) = 2x - \frac{2^3 x^3}{3!} + \cdots + \frac{(-1)^{n+1} 2^{2n+1} x^{2n+1}}{(2n+1)!} + \frac{(-1)^n \cos(2t) (2x)^{2n+3}}{(2n+3)!}, f\left(\frac{1}{2}\right) \sim 0.8415$$

$$5 \quad f(x) = x - \frac{4x^3}{3!} + \frac{4^2 x^5}{5!} - \cdots + \frac{(-1)^n 4^n x^{2n+1}}{(2n+1)!} + \frac{(-1)^{n+1} 4^{n+1} (\cos^2 t - \sin^2 t) x^{2n+3}}{(2n+3)!},$$

$$f\left(\frac{1}{2}\right) \sim 0.4207$$

$$7 \quad f(x) = 2^{-3} + 3 \cdot 2^{-6} x + \cdots + \frac{(-1)^n (1 \cdot 3 \cdots (2n+1)) 2^{-3(n+1)} x^n}{n!} + \frac{(-1)^{n+1} (1 \cdot 3 \cdots (2n+3)) (4+t)^{(-5/2-n)} x^{n+1}}{2^{n+1} (n+1)!}, f\left(\frac{1}{2}\right) \sim 0.1048$$

$$9 \quad x + \frac{x^3}{3}, 0.346 \quad 11 \quad x + \frac{x^3}{6}, 0.340 \quad 13 \quad x - \frac{x^2}{2}, 0.28 \quad 15 \quad x + \frac{x^3}{3}, 0.346$$

$$17 \quad \frac{x^2}{2} - \frac{x^3}{6}, 0.049$$

$$19 \quad e^x = e^2 + e^2(x-2) + \frac{e^2(x-2)^2}{2!} + \cdots + \frac{e^2(x-2)^n}{n!} + \frac{e^t(x-2)^{n+1}}{(n+1)!}$$

$$21 \quad x^p = 1 + p(x-1) + \frac{p(p-1)(x-1)^2}{2} + \cdots + \frac{p(p-1)\cdots(p-n+1)(x-1)^n}{n!} + \frac{p(p-1)\cdots(p-n)t^{p-n-1}(x-1)^{n+1}}{(n+1)!}$$

### Section 9.11

$$1 \quad 96 \quad 3 \quad 6 \quad 5 \quad 1008 \quad 7 \quad \sum_{n=0}^{\infty} x^n / (2^n n!), r = \infty$$

$$9 \quad 1 + x + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} (1 \cdot 3 \cdot 5 \cdots (2n-3)) x^n}{n!}, r = \frac{1}{2}$$

$$11 \quad \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}, \text{ converges to } \cos \sqrt{x} \text{ for } x \geq 0 \quad 13 \quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}, r = \infty$$

$$15 \quad 1 - \frac{x^2}{2} + \sum_{n=2}^{\infty} \frac{(1 \cdot 3 \cdot 5 \cdots (2n-3)) x^{2n}}{2^n n!}, r = 1 \quad 17 \quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+4}}{(6n+4)(2n+1)!}, r = \infty$$

$$19 \quad \sum_{n=0}^{\infty} \frac{x^{4n+1}}{(4n+1)(2n+1)!}, r = \infty \quad 21 \quad x + \sum_{n=1}^{\infty} \frac{\frac{1}{3}(\frac{1}{3}-1) \cdots (\frac{1}{3}-n+1) x^{2n+1}}{n!(2n+1)}, r = 1$$

$$23 \quad x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdots (2n-1) x^{2n+1}}{2^n n! (2n+1)^2}, r = 1 \quad 25 \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-1)^n}{n}, r = 1$$

### Extra Problems for Chapter 9

$$1 \text{ converges to } 1 \quad 3 \text{ converges to } 1 \quad 5 \text{ diverges to } \infty \quad 7 \text{ diverges}$$

$$9 \text{ converges to } 28\frac{4}{5} \quad 11 \text{ converges to } 42 \quad 13 \text{ diverges} \quad 15 \text{ diverges}$$

$$17 \text{ converges} \quad 19 \text{ converges} \quad 21 \text{ diverges} \quad 23 \text{ converges} \quad 25 \text{ converges}$$

$$27 \text{ converges} \quad 29 \text{ diverges} \quad 31 \quad 1/2 \quad 33 \quad 1 \quad 35 \quad \sqrt{e}$$

$$37 \quad f'(x) = \sum_{n=1}^{\infty} n^{n+1} (n+1) 2^n x^{n-1}, \int_0^x f(t) dt = \sum_{n=1}^{\infty} n^n (n+1) 2^n x^{n+1}, r = \frac{1}{2}$$

$$39 \quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(2n+1)(4n+1)}, r = 1 \quad 41 \quad -0.006 \quad 43 \quad 0.646$$

$$45 \quad x + \sum_{n=1}^{\infty} \frac{2^n (-1)^n (2 \cdot 5 \cdot 8 \cdots (3n-1)) x^{2n+1}}{3^n (2n+1)n!}, r = \frac{1}{\sqrt{2}} \quad 47 \quad e^{50}$$

### Section 10.1

$$1 \quad i + 2j \quad 3 \quad -3i - 4j \quad 5 \quad (2, -4) \quad 7 \quad (-1, 12) \quad 9 \quad \mathbf{A} + \mathbf{B} = -3i + j$$

$$11 \quad \mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{j} \quad 13 \quad 3\mathbf{A} = 3i - 6j \quad 15 \quad \mathbf{B} - \mathbf{A} = -5i + 5j$$

- 17  $\mathbf{A} - 2\mathbf{B} + 3\mathbf{C} = 18\mathbf{i} - 8\mathbf{j}$  19  $|\mathbf{B}| = 5$  21  $|\mathbf{A} - \mathbf{B}| = 5\sqrt{2}$  23  $|\mathbf{6A}| = 6\sqrt{5}$   
 25  $\frac{13}{4}\mathbf{i} - \frac{5}{4}\mathbf{j}$  27  $-\frac{4}{3}\mathbf{i} + \frac{2}{3}\mathbf{j}, (-\frac{4}{3}, \frac{2}{3})$  29  $\arccos(-\frac{2\sqrt{5}}{5})$  31  $\arccos(-\frac{4}{5})$   
 33  $7\mathbf{i} + 3\mathbf{j}$  35  $8\mathbf{i} - 2\mathbf{j}$  37  $-4\mathbf{i} - 6\mathbf{j}$  39 5 41  $4\mathbf{i} + 3\mathbf{j}$  43  $10\mathbf{i} + 2\mathbf{j}$   
 45  $-2\sqrt{2}\mathbf{i} + 2\sqrt{2}\mathbf{j}$  47  $6\mathbf{i} + 8\mathbf{j}$  49  $2\mathbf{i} - \mathbf{j}, 2\mathbf{i} + 2\mathbf{j}, -5\mathbf{i} + 2\mathbf{j}, \mathbf{i} - 3\mathbf{j}$

## Section 10.2

- 1  $\mathbf{X} = 3\mathbf{i} - \mathbf{j} + t(-\mathbf{i} + \mathbf{j})$  3  $\mathbf{X} = 3\mathbf{i} + 4\mathbf{j} + t(-2\mathbf{i} + 5\mathbf{j})$  5  $\mathbf{X} = \mathbf{i} + 4\mathbf{j} + t(\mathbf{i} - 5\mathbf{j})$   
 7  $\mathbf{X} = 2\mathbf{i} + 5\mathbf{j} + t\mathbf{j}$  9  $\mathbf{X} = 2\mathbf{j} + t(\mathbf{i} + 5\mathbf{j})$  11  $\mathbf{X} = 3\mathbf{j} + t\mathbf{i}$   
 13  $\mathbf{X} = 6\mathbf{i} + 5\mathbf{j} + t(\mathbf{i} - 3\mathbf{j})$  15  $y = -2x + 10$  17  $y = 3$  19 no 21 yes  
 23 yes 25  $(-2, 3)$  27  $(2, 10)$  29  $(4, 3)$

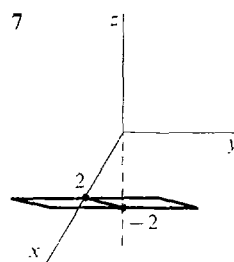
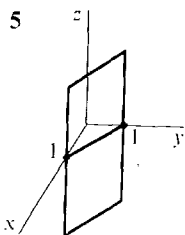
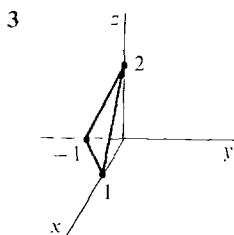
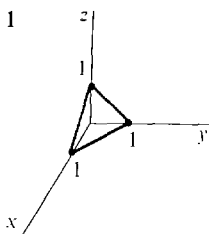
## Section 10.3

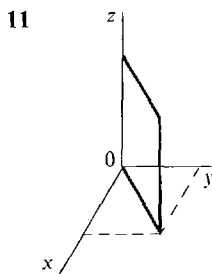
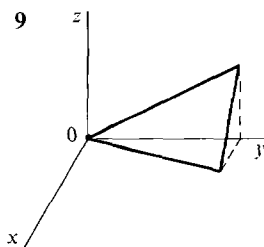
- 1  $5\mathbf{i} - \mathbf{j} + 7\mathbf{k}$  3  $-\mathbf{k}$  5  $(0, 0, 0)$  7  $3\mathbf{i} + \mathbf{j} - 4\mathbf{k}$  9  $\mathbf{i} + 5\mathbf{j} - 8\mathbf{k}$   
 11  $-2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$  13 3 15  $\sqrt{26}$  17  $\arccos(\frac{8}{21})$  19 0  
 21  $\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}, (\frac{1}{3}, -\frac{2}{3}, \frac{2}{3})$  23  $-3\mathbf{i} + 3\mathbf{j} + 3\sqrt{2}\mathbf{k}$  25  $\frac{2}{3}, -\frac{2}{3}$  27  $\frac{3}{10}\mathbf{i} - \frac{1}{2}\mathbf{j} + \frac{1}{10}\mathbf{k}$   
 29  $125\mathbf{i} + 250\mathbf{j} + 625\mathbf{k}$  31  $\mathbf{X} = \mathbf{i} + 3\mathbf{j} + \mathbf{k} + t(\mathbf{i} - \mathbf{k})$   
 33  $\mathbf{X} = -\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + t(-\mathbf{i} - 7\mathbf{j} + 3\mathbf{k})$  35  $\mathbf{X} = t(-3\mathbf{i} + 4\mathbf{k})$

## Section 10.4

- 1 (a) 20 (b)  $\mathbf{A} \parallel \mathbf{B}$  (c) 1 3 (a) 0 (b)  $\mathbf{A} \perp \mathbf{B}$  (c) 0  
 5 (a) 24 (b) neither (c)  $12/37$  7 (a)  $-92$  (b) neither (c)  $-92/117$   
 9 (a) 0 (b)  $\mathbf{A} \perp \mathbf{B}$  (c) 0 11 (a)  $8\sqrt{5}$  (b)  $\mathbf{A} \parallel \mathbf{B}$  (c) 1 13 41  
 15  $3\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$  17  $200\sqrt{2}$  19  $\mathbf{i} - \mathbf{j}$  21  $-2\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$  23  $-\mathbf{i} + \mathbf{j} + \mathbf{k}$   
 23  $-\mathbf{i} - \mathbf{j} + \mathbf{k}$  27  $\frac{4}{3}\mathbf{i} + \frac{2}{3}\mathbf{j}$  29  $\mathbf{i} - \mathbf{j}$  31  $\arccos(\frac{1}{3})$  33  $\pi/3$

## Section 10.5





- 13 (a)  $\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$  (b)  $\mathbf{i} + 2\mathbf{j}$  (c)  $-3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$  (d)  $\mathbf{i} + 6\mathbf{k}$  (e)  $\mathbf{j}$  (f)  $-\mathbf{j} + \mathbf{k}$   
 15  $y + 2z = -9$  17  $x + y + 2z = 0$  19  $y - z = -1$   
 21  $4x + 20y - 5z = 20$  23  $x + 2y + z = 9$  25  $2x + y + 3z = 13$   
 27  $x + y - 2z = 20$  29  $x - y - 3z = 3$  31  $24x - 19y + 4z = 57$  33 parallel  
 35 neither 37  $\mathbf{X} = 5\mathbf{i} + 3\mathbf{j} - \mathbf{k} + t(\mathbf{i} - \mathbf{j} + 3\mathbf{k})$  39  $\mathbf{X} = -\mathbf{i} + \mathbf{k} + t(3\mathbf{i} - \mathbf{j} - 2\mathbf{k})$   
 41  $\mathbf{X} = \mathbf{j} - \mathbf{k} + t(-\mathbf{i} + \mathbf{j} + \mathbf{k})$  43  $(3, 1, 1)$  45  $(-\frac{4}{3}, 0, \frac{8}{3})$  47  $(-2, 1, 2)$   
 49  $(1, 1, 3)$

## Section 10.6

- 1  $\mathbf{X} = 3t\mathbf{i} + 9t^2\mathbf{j}$  3  $\mathbf{X} = \frac{1}{\sqrt{5}}t^3\mathbf{i} + \frac{2}{\sqrt{5}}t^3\mathbf{j}$  5  $\mathbf{X} = (1/\sqrt{t^2 + 1})(\mathbf{i} + \mathbf{j})$   
 7  $\mathbf{X} = \frac{\cos t + \cos(3t)}{2}\mathbf{i} + \frac{\sin t + \sin(3t)}{2}\mathbf{j}$   
 9  $\mathbf{X} = (4 \cos(t/3) - \cos(4t/3))\mathbf{i} + (4 \sin(t/3) - \sin(4t/3))\mathbf{j}$   
 11  $\mathbf{X} = (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j}$  13  $\mathbf{X} = \cos t\mathbf{i} + \sin t\mathbf{j} + t^2\mathbf{k}$   
 15  $\mathbf{X} = \mathbf{i} + \left(2 + \frac{t^2}{\sqrt{2t^4 - 2t^2 + 1}}\right)\mathbf{j} + \left(1 + \frac{t^2 - 1}{\sqrt{2t^4 - 2t^2 + 1}}\right)\mathbf{k}$   
 17  $\mathbf{X} = \frac{1}{2}(\cos t + \cos(2t))\mathbf{i} + \frac{1}{2}\sin t\mathbf{j} + \frac{1}{2}\sin(2t)\mathbf{k}$   
 19  $\mathbf{X} = (-\frac{5}{3} + \frac{2}{3}\cos t - \frac{1}{3}\sin t)\mathbf{i} + (-\frac{5}{3} - \frac{1}{3}\cos t + \frac{2}{3}\sin t)\mathbf{j} + (\frac{1}{3} - \frac{1}{3}\cos t - \frac{1}{3}\sin t)\mathbf{k}$   
 21  $\mathbf{X} = (t - t \cos^2 t - 2t \sin t \cos t)\mathbf{i} + (2t - t \sin t \cos t - 2t \sin^2 t)\mathbf{j} + 3t\mathbf{k}$   
 23  $\mathbf{P}(t) = 2t\mathbf{i} + \frac{1}{2t(t+1)}\mathbf{j} + (t+1)\mathbf{k}$

## Section 10.7

- 1  $5 \cos t\mathbf{i} - 5 \sin t\mathbf{j}$  3  $-e^t \sin(e^t)\mathbf{i} + e^t \cos(e^t)\mathbf{j}$  5  $-6\mathbf{i} + (6/t)\mathbf{j} - 6e^t\mathbf{k}$   
 7  $2 \cos 2t$  9  $t/\sqrt{1+t^2}$  11  $\mathbf{i} + \mathbf{j} + \mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$   
 13  $\mathbf{V} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ ,  $|\mathbf{V}| = \sqrt{29}$ ,  $\mathbf{A} = \mathbf{0}$   
 15  $\mathbf{V} = -\sin t\mathbf{i} + \cos t\mathbf{j} + \mathbf{k}$ ,  $|\mathbf{V}| = \sqrt{2}$ ,  $\mathbf{A} = -\cos t\mathbf{i} - \sin t\mathbf{j}$   
 17  $\mathbf{V} = -e^t \sin(e^t)\mathbf{i} + e^t \cos(e^t)\mathbf{j}$ ,  $|\mathbf{V}| = e^t$ ,  
 $\mathbf{A} = (-e^t \sin e^t - e^{2t} \cos e^t)\mathbf{i} + (e^t \cos e^t - e^{2t} \sin e^t)\mathbf{j}$   
 19  $\mathbf{V} = 2t\mathbf{i} + 4t\mathbf{j} - 2t\mathbf{k}$ ,  $|\mathbf{V}| = 2\sqrt{6}|t|$ ,  $\mathbf{A} = 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$   
 21  $\mathbf{S} = t \cos t\mathbf{i} + t \sin t\mathbf{j}$ ,  $\mathbf{V} = (-t \sin t + \cos t)\mathbf{i} + (t \cos t + \sin t)\mathbf{j}$ ,  $|\mathbf{V}| = \sqrt{1+t^2}$ ,  
 $\mathbf{A} = (-t \cos t - 2 \sin t)\mathbf{i} + (-t \sin t + 2 \cos t)\mathbf{j}$   
 23  $\mathbf{S} = \cos(t^2)\mathbf{i} + \sin(t^2)\mathbf{j}$ ,  $\mathbf{V} = -2t \sin(t^2)\mathbf{i} + 2t \cos(t^2)\mathbf{j}$ ,  $|\mathbf{V}| = |2t|$ ,  
 $\mathbf{A} = (-2 \sin(t^2) - 4t^2 \cos(t^2))\mathbf{i} + (2 \cos(t^2) - 4t^2 \sin(t^2))\mathbf{j}$   
 25  $\mathbf{V} = (e^x/\sqrt{1+e^{2x}})\mathbf{i} + (e^{2x}/\sqrt{1+e^{2x}})\mathbf{j}$ ,  $|\mathbf{V}| = e^x$ ,  
 $\mathbf{A} = [-e^{3x}(1+e^{2x})^{-3/2} + e^x(1+e^{2x})^{-1/2}]\mathbf{i} + [-e^{4x}(1+e^{2x})^{-3/2} + 2e^{2x}(1+e^{2x})^{-1/2}]\mathbf{j}$   
 27  $2\pi$  29  $5$  31  $2\sqrt{3} + \sqrt{2} \ln(\sqrt{3} + \sqrt{2})$  33  $\frac{3}{2} + \ln 2$   
 35  $\mathbf{F}(t) = (\frac{1}{2}t^2 + \frac{1}{2})\mathbf{i} + (\frac{1}{3}t^3 + \frac{5}{3})\mathbf{j} + (\frac{1}{4}t^4 + \frac{11}{4})\mathbf{k}$

37  $F(t) = \ln|t - 1|\mathbf{i} + (\ln|t - 2| - \ln 2)\mathbf{j} + (\ln|t - 3| - \ln 3)\mathbf{k}$

39  $\mathbf{S} = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$

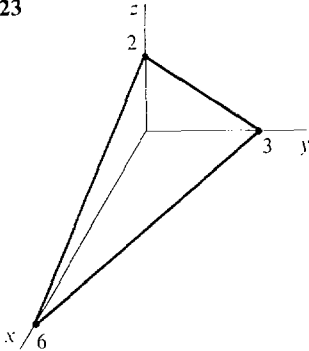
## Section 10.8

- 1 infinitesimal    3 infinite    5 finite    7 infinite    9 infinite  
 11 infinitesimal (zero)    13 infinite    15 infinite    17 infinitesimal    19 infinite  
 21  $-\sin x\mathbf{i} + \cos x\mathbf{j}$     23  $10\mathbf{i} - 20\mathbf{j} + 5\mathbf{k}$     25  $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$     27 0  
 29  $(1/|A|)A \cdot \mathbf{U}$  if  $A \neq 0$ , 1 if  $A = 0$     31 (a) no (b) no    33 (a) yes (b) no  
 35 (a) yes (b) no    37 (a) no (b) no    39 (a) yes (b) no

## Extra Problems for Chapter 10

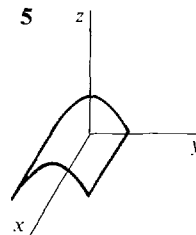
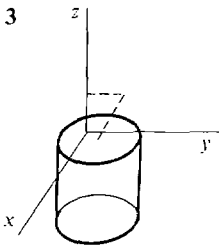
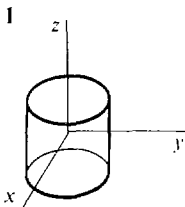
- 1  $5\mathbf{i} - 12\mathbf{j}$     3  $-3\mathbf{i} - 3\mathbf{j}$     5  $9\mathbf{i} + 11\mathbf{j}$     7  $\mathbf{X} = -\mathbf{i} + \mathbf{j} + t(4\mathbf{i} - 3\mathbf{j})$     9  $(2, 5/2)$   
 11  $(1/\sqrt{105}, -10/\sqrt{105}, 2/\sqrt{105})$     13  $100\mathbf{i} - 500\mathbf{j} + 300\mathbf{k}$   
 15  $\mathbf{X} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + t\mathbf{k}$     17  $\mathbf{A} \perp \mathbf{B}$     19 -20    21  $-\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

23

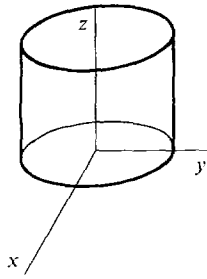


- 25  $-x - y + 2z = 0$     27  $(-1, -2, 4)$   
 29  $\mathbf{S} = \sin t \cos t\mathbf{i} + \sin^2 t\mathbf{j} + \cos t\mathbf{k}$ ,  $\mathbf{S} = -\sin t \cos t\mathbf{i} - \sin^2 t\mathbf{j} + \cos t\mathbf{k}$   
 31  $\mathbf{V} = \cos(2t)\mathbf{i} + \sin(2t)\mathbf{j} - \sin t\mathbf{k}$  or  $\mathbf{V} = -\cos(2t)\mathbf{i} - \sin(2t)\mathbf{j} - \sin t\mathbf{k}$ ,  $|\mathbf{V}| = \sqrt{1 + \sin^2 t}$ ,  
 $\mathbf{A} = -2 \sin(2t)\mathbf{i} + 2 \cos(2t)\mathbf{j} - \cos t\mathbf{k}$  or  $\mathbf{A} = 2 \sin(2t)\mathbf{i} - 2 \cos(2t)\mathbf{j} - \cos t\mathbf{k}$   
 33  $\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{1}{3}\mathbf{k} + t(-\mathbf{i} - \frac{1}{2}\mathbf{j} - \frac{1}{3}\mathbf{k})$   
 35  $\mathbf{S} = (-\cos e^t + \cos 1)\mathbf{i} + (\sin e^t - \sin 1)\mathbf{j} + (e^t - 1)\mathbf{k}$

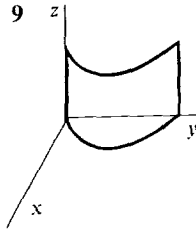
## Section 11.1



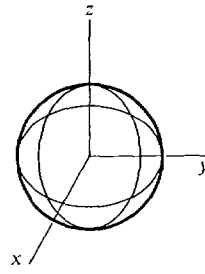
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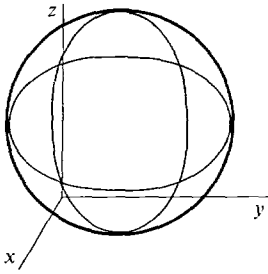
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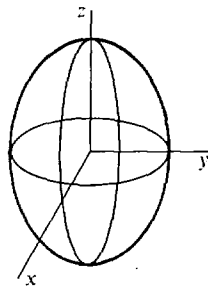
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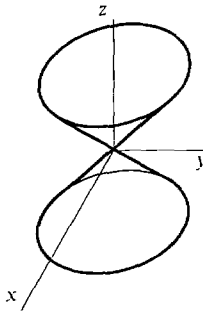
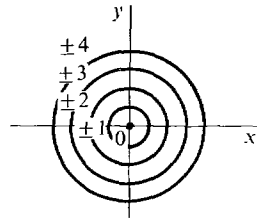
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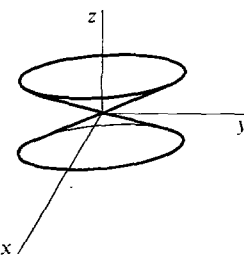
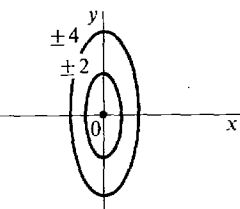
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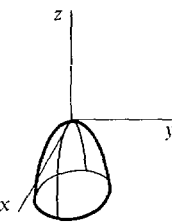
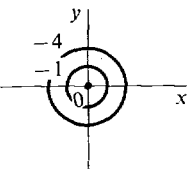
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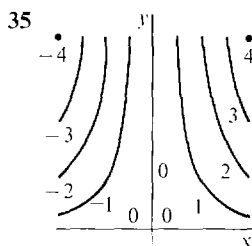
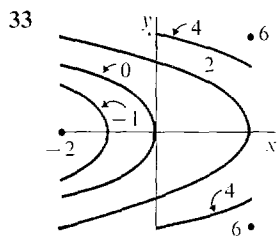
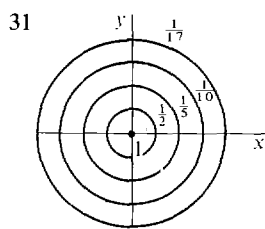
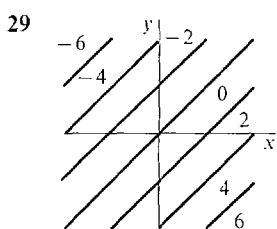
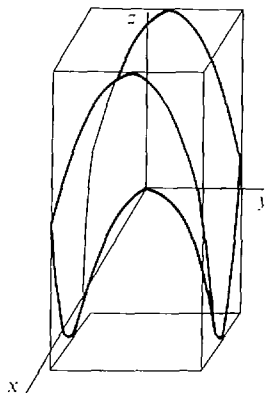
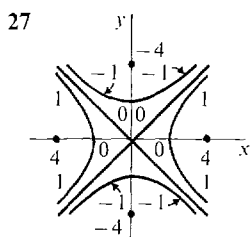
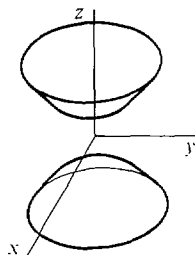
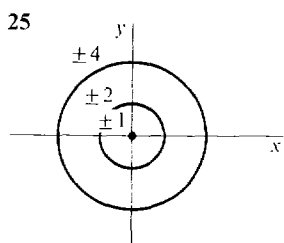
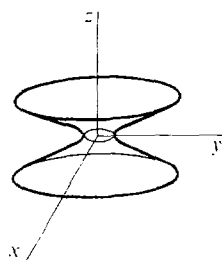
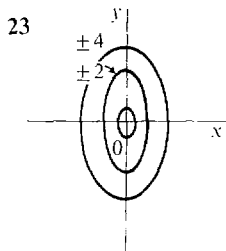


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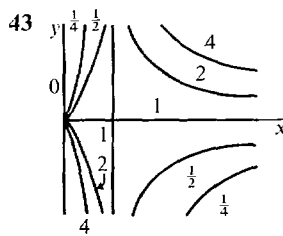
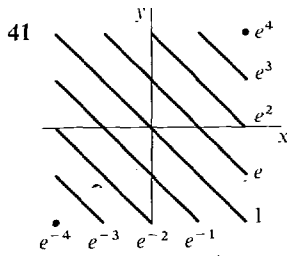
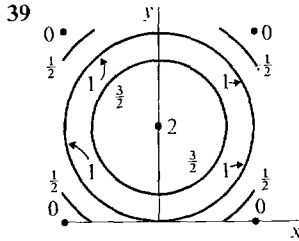
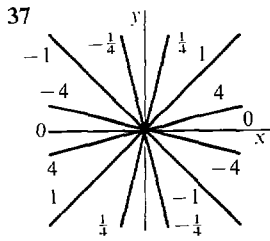


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## Section 11.2

- 1 all  $(x, y)$     3 all  $(x, y)$     5  $y \neq -x$     7  $y \neq -2$     9  $x \neq 2$  and  $y \neq -1$   
 11  $x + y > 0$     13  $x > y$     15  $x > 0$     17  $x^2 > y$     19  $x > 0$  and  $y > 0$   
 21  $x^2 > y$     23  $x > 0$  and  $y > 0$  and  $x \neq 1/y$     25  $x > 0$  and  $y > 0$   
 27  $x \neq 4y$  and  $y^2 > x$     29  $\cos x + y > 0$     31  $x \neq y$     33  $x > y$  and  $y \neq z$   
 35  $(x, y, z) \neq (0, 0, 0)$     37  $x + y > 0$  and  $z \neq 0$     43  $e$     45  $\infty$

## Section 11.3

- 1  $\partial z/\partial x = 4, \partial z/\partial y = -3$     3  $\partial z/\partial x = y^2 + 3x^2y, \partial z/\partial y = 2xy + x^3$   
 5  $\frac{\partial z}{\partial x} = -\frac{2x}{(x^2 + y^2)^2}, \frac{\partial z}{\partial y} = -\frac{2y}{(x^2 + y^2)^2}$     7  $f_x(x, y) = y, f_y(x, y) = x$   
 9  $f_x(x, y) = a, f_y(x, y) = b$     11  $f_x(x, y) = 2xe^{x^2-y^2}, f_y(x, y) = -2ye^{x^2-y^2}$   
 13  $f_x(x, y) = \frac{1}{2\sqrt{x+2y}}, f_y(x, y) = \frac{1}{\sqrt{x+2y}}$     15  $\partial z/\partial x = yx^{y-1}, \partial z/\partial y = x^y \ln x$   
 17  $\partial z/\partial x = 1/x, \partial z/\partial y = 1/y$   
 19  $\frac{\partial z}{\partial x} = -\frac{\ln y}{x(\ln x)^2}, \frac{\partial z}{\partial y} = \frac{1}{y \ln x}$     21  $\frac{\partial z}{\partial x} = \frac{2xy}{\sqrt{1-x^4y^2}}, \frac{\partial z}{\partial y} = \frac{x^2}{\sqrt{1-x^4y^2}}$   
 23  $\frac{\partial w}{\partial x} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}, \frac{\partial w}{\partial y} = \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \frac{\partial w}{\partial z} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$   
 25  $f_x(x, y, z) = a, f_y(x, y, z) = b, f_z(x, y, z) = c$   
 27  $\partial w/\partial x = -z \sin x, \partial w/\partial y = z \cos y, \partial w/\partial z = \cos x + \sin y$     29  $f_x(1, 2) = 4, f_y(1, 2) = 4$   
 31  $f_x(-1, 1) = -1, f_y(-1, 1) = 1$     33  $\frac{\partial z}{\partial x}(0, 2) = 2, \frac{\partial z}{\partial y}(0, 2) = 0$   
 35  $\frac{\partial z}{\partial x}(1, 0) = e, \frac{\partial z}{\partial y}(1, 0) = 0$     37  $\frac{\partial z}{\partial x}(2, 3) = -\frac{4}{961}, \frac{\partial z}{\partial y}(2, 3) = -\frac{27}{961}$   
 39  $f_x(1, 2, 3) = 2, f_y(1, 2, 3) = 4, f_z(1, 2, 3) = 6$     41  $1/\sqrt{5}$     43  $1/3$     45  $b = 2, c = 1$   
 47  $C_x(x, y) = 3 - \frac{1}{2}\sqrt{y/x}, C_y(x, y) = 4 - \frac{1}{2}\sqrt{x/y}$

## Section 11.4

- 1  $\Delta z = 3 \Delta x - 2 \Delta y, dz = 3 dx - 2 dy$   
 3  $\Delta z = 2xy^2 \Delta x + 2x^2y \Delta y + y^2 \Delta x^2 + 4xy \Delta x \Delta y + x^2 \Delta y^2 + 2x \Delta x \Delta y^2 + 2y \Delta x^2 \Delta y + \Delta x^2 \Delta y^2, dz = 2xy^2 dx + 2x^2y dy$   
 5  $\Delta z = -\frac{x \Delta y + y \Delta x + \Delta x \Delta y}{(x + \Delta x)(y + \Delta y)xy}, dz = -\frac{dx}{x^2y} - \frac{dy}{xy^2}$   
 7  $\Delta z = e^{3x-4y}(e^{3\Delta x-4\Delta y} - 1), dz = e^{3x-4y}(3 dx - 4 dy)$   
 9  $\Delta z = \cos(x + \Delta x) \sin(y + \Delta y) - \cos x \sin y, dz = -\sin x \sin y dx + \cos x \cos y dy$   
 11  $\Delta z = x \ln(1 + (\Delta y/y)) + \Delta x \ln(y + \Delta y), dz = \ln y dx + (x/y) dy$   
 13  $\Delta w = \Delta x + 2 \Delta y + 3 \Delta z, dw = dx + 2 dy + 3 dz$   
 15  $\Delta w = y \Delta x + (x + z) \Delta y + y \Delta z + \Delta x \Delta y + \Delta y \Delta z, dw = y dx + (x + z) dy + y dz$   
 17  $\Delta z = dz + \Delta x \Delta x + \Delta y \Delta y$     19  $\Delta z = dz + (y \Delta x + 2x \Delta y + \Delta x \Delta y) \Delta x + 0 \Delta y$   
 21  $\Delta z = dz + \frac{\Delta y}{y(y + \Delta y)} \Delta x + \frac{x \Delta y}{y^2(y + \Delta y)} \Delta y$     23  $z = 4x + 4y - 6$   
 25  $z = 4x + 4y - 2$     27  $z = \frac{1}{2}x + \frac{1}{2}y + 1$     29  $z = 6e^3x + e^3y - 8e^3$   
 31  $z = \frac{\sqrt{2}}{4}x + \frac{\sqrt{6}}{4}y + \frac{\sqrt{6}}{4} - \pi\left(\frac{\sqrt{2}}{12} + \frac{\sqrt{6}}{16}\right)$     33  $z = x - 2$     35  $z = 1$   
 37  $x - 2y + 2z = 9$     39  $x + y - z = 1$

## Section 11.5

- 1  $2e^{2t} + 2e^{-2t}$     3  $-\frac{\cos(t/a) + \cos(t/b)}{[a \sin(t/a) + b \sin(t/b)]^2}$     5  $\frac{2y}{x \ln y} - \frac{2x \ln x}{y(\ln y)^2}$   
 7  $(t + 1)^{1/t} \left[ \frac{1}{t(t + 1)} - \frac{\ln(t + 1)}{t^2} \right]$     9  $(\sin t)^{\cos t} \left[ \frac{\cos^2 t}{\sin t} - \sin t \ln(\sin t) \right]$   
 11  $\frac{2t}{(t^2 - 1) \ln(t^2 + 1)} - \frac{2t \ln(t^2 - 1)}{(t^2 + 1)[\ln(t^2 + 1)]^2}$     13  $3\sqrt{1 - t^4} - 2\sqrt{1 - t^3}$   
 15  $\frac{\partial z}{\partial s} = 3s^2 \cos^3 t, \frac{\partial z}{\partial t} = -3s^2 \sin t \cos^2 t$     17  $\frac{\partial z}{\partial s} = \frac{2s}{s^2 - t^2}, \frac{\partial z}{\partial t} = -\frac{2t}{s^2 - t^2}$   
 19  $\frac{\partial z}{\partial s} = -\frac{a}{(s + t)^2} - \frac{b}{(s - t)^2}, \frac{\partial z}{\partial t} = -\frac{a}{(s + t)^2} + \frac{b}{(s - t)^2}$     21  $b \frac{\partial z}{\partial x} = abf'(ax + by) = a \frac{\partial z}{\partial y}$   
 25  $\frac{dw}{dt} = e^t \cos(\sqrt{t}) - e^{-t} \sin(\sqrt{t}) + \frac{1}{2\sqrt{t}} (-e^t \sin(\sqrt{t}) + e^{-t} \cos(\sqrt{t}))$   
 27  $\frac{dz}{dt} = \frac{x(dx/dt) + y(dy/dt)}{\sqrt{x^2 + y^2}}$  or  $\frac{dz}{dt} = \frac{x}{z} \frac{dx}{dt} + \frac{y}{z} \frac{dy}{dt}$   
 29  $\frac{dz}{dt} = x^y \left( \frac{y}{x} \frac{dx}{dt} + \ln x \frac{dy}{dt} \right)$  or  $\frac{dz}{dt} = \frac{zy}{x} \frac{dx}{dt} + z \ln x \frac{dy}{dt}$   
 31  $\frac{\partial z}{\partial s} = af'(u), \frac{\partial z}{\partial t} = bf'(u)$     33  $\frac{\partial z}{\partial s} = z \frac{\partial u}{\partial s}, \frac{\partial z}{\partial t} = z \frac{\partial u}{\partial t}$   
 35  $\frac{\partial z}{\partial s} = g'(s)h(t), \frac{\partial z}{\partial t} = g(s)h'(t)$     37  $-2$     39  $-3$     41  $2$     43  $1/25$

## Section 11.6

- 1  $\frac{\partial z}{\partial x} = 3, \frac{dz}{dx} = 3 - 4e^x$     3  $\frac{\partial z}{\partial x} = -\sin x, \frac{dz}{dx} = -\sin x + 3 \cos(3x)$   
 5  $\frac{\partial z}{\partial x} = yx^{y-1}, \frac{dz}{dx} = x^x(1 + \ln x)$     7  $\frac{\partial z}{\partial x} = \frac{y}{1 + x^2y^2}, \frac{dz}{dx} = \frac{e^{-x}(1 - x)}{1 + x^2e^{-2x}}$     9  $\frac{y + x}{y - x}$   
 11  $-\frac{2x + 2y^3}{6xy^2 + 1}$     13  $-\frac{1 + y \cos(xy)}{x \cos(xy)}$     15 slope =  $-\frac{3}{4}, 4y = -3x + 14$   
 17 slope = 5,  $y = 5x - 6$     19 slope =  $-1, y = -x + 2$     21 slope = 1,  $y = x$   
 23  $\frac{\partial w}{\partial x}(x, y) = 15, \frac{\partial w}{\partial y}(x, y) = -34$

$$25 \quad \frac{\partial w}{\partial x}(x, y) = \frac{10x - 6y}{\sqrt{10x^2 - 12xy + 5y^2}}, \quad \frac{\partial w}{\partial y}(x, y) = \frac{-6x + 5y}{\sqrt{10x^2 - 12xy + 5y^2}}$$

$$27 \quad 6x - 5y + 4z = -21 \quad 29 \quad x + y + z = 3 \quad 31 \quad x + y + z = 0$$

$$33 \quad \frac{\partial z}{\partial x} = -\frac{xy}{z}, \quad \frac{\partial z}{\partial y} = -\frac{x^2}{2z} \quad 35 \quad \frac{\partial z}{\partial x} = \frac{\cos(xy)}{\sin(yz)}, \quad \frac{\partial z}{\partial y} = \frac{x \cos(xy) - z \sin(yz)}{y \sin(yz)}$$

$$37 \quad \frac{\partial z}{\partial x} = -\frac{z}{3x}, \quad \frac{\partial z}{\partial y} = -\frac{2z}{3y} \quad 39 \quad \frac{dz}{dx} = y - \frac{x}{1 + \sqrt{x}}$$

$$41 \quad \frac{dw}{dx} = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} \frac{dy}{dx} + \frac{\partial w}{\partial z} \frac{dz}{dx}$$

## Section 11.7

- 1 max = 3 at  $(-1, -1)$  and  $(1, 1)$ ; min = 0 at  $(0, 0)$   
 3 max = 48 at  $(3, 3)$ ; min =  $-6$  at  $(1, -2)$     5 max = 4 at  $(2, 1)$ ; min =  $-5$  at  $(2, 4)$   
 7 max = 2 at  $(\pi/2, \pi/2)$ ; min = 0 at  $(0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$   
 9 max = 1 at  $(-1, 1)$  and  $(1, 1)$ ; min =  $-\frac{1}{4}$  at  $(0, \frac{1}{2})$   
 11 max =  $\frac{1}{4}$  at  $(\frac{1}{2}, 0, \frac{1}{2})$  and  $(\frac{1}{2}, 1, -\frac{1}{2})$ ; min =  $-\frac{1}{4}$  at  $(0, \frac{1}{2}, \frac{1}{2})$  and  $(1, \frac{1}{2}, -\frac{1}{2})$   
 13 max =  $1 + \sqrt{2}$  at  $(\sqrt{2}/2, \sqrt{2}/2, 1)$ ; min =  $-\frac{1}{2}$  at  $(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$   
 15 max = 3 at  $(-1, -1, 1), (-1, 1, 1), (1, -1, 1), (1, 1, 1)$ ; min = 0 at  $(0, 0, 0)$   
 17 no max; min =  $-4$  at  $(-2, 0)$     19 no max; no min  
 21 no max; min = 6 at  $(\frac{1}{2}, 4)$     23 max = 1 at  $(0, 0)$ ; no min    25 no max, no min  
 27  $x = 4, y = 2, z = 2$     29 6 in.  $\times$  6 in.  $\times$  12 in.    31  $2\sqrt{3}/3$     33  $(\frac{5}{3}, \frac{10}{3}, -\frac{5}{3})$   
 35  $(1, 1, 1), (1, -1, -1), (-1, 1, -1), (-1, -1, 1)$   
 39  $\frac{2}{3}(\frac{2}{3}V)^{1/3}$  in.  $\times$   $\frac{2}{3}(\frac{2}{3}V)^{1/3}$  in.  $\times$   $(\frac{2}{3}V)^{1/3}$  in.    41  $x = 100, y = 200, P(100, 200) = 50,000$

## Section 11.8

$$1 \quad \frac{\partial^2 z}{\partial x^2} = 2, \quad \frac{\partial^2 z}{\partial y^2} = 4, \quad \frac{\partial^2 z}{\partial x \partial y} = 0 \quad 3 \quad \frac{\partial^2 z}{\partial x^2} = 2a, \quad \frac{\partial^2 z}{\partial y^2} = 2c, \quad \frac{\partial^2 z}{\partial x \partial y} = b$$

$$5 \quad \frac{\partial^2 z}{\partial x^2} = e^{x+y}(x+2), \quad \frac{\partial^2 z}{\partial y^2} = xe^{x+y}, \quad \frac{\partial^2 z}{\partial x \partial y} = e^{x+y}(x+1)$$

$$7 \quad \frac{\partial^2 z}{\partial x^2} = -\frac{a^2}{(ax+by)^2}, \quad \frac{\partial^2 z}{\partial y^2} = -\frac{b^2}{(ax+by)^2}, \quad \frac{\partial^2 z}{\partial x \partial y} = -\frac{ab}{(ax+by)^2}$$

$$9 \quad \frac{\partial^2 z}{\partial x^2} = a(a-1)x^{a-2}y^b, \quad \frac{\partial^2 z}{\partial y^2} = b(b-1)x^a y^{b-2}, \quad \frac{\partial^2 z}{\partial x \partial y} = abx^{a-1}y^{b-1}$$

$$11 \quad \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial z^2} = \frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial x \partial z} = \frac{\partial^2 w}{\partial y \partial z} = -\frac{1}{4}(x+y+z)^{-3/2}$$

$$13 \quad \frac{\partial^3 z}{\partial x^3} = 12y^2, \quad \frac{\partial^3 z}{\partial y^3} = -36x^2, \quad \frac{\partial^3 z}{\partial x^2 \partial y} = 24xy - 36y^2, \quad \frac{\partial^3 z}{\partial x \partial y^2} = 12x^2 - 72xy$$

$$15 \quad \frac{\partial^3 z}{\partial x^3} = a^3 e^{ax+by}, \quad \frac{\partial^3 z}{\partial y^3} = b^3 e^{ax+by}, \quad \frac{\partial^3 z}{\partial x^2 \partial y} = a^2 b e^{ax+by}, \quad \frac{\partial^3 z}{\partial x \partial y^2} = ab^2 e^{ax+by}$$

$$17 \quad \frac{\partial^2 z}{\partial \theta^2} = r^2 \sin^2 \theta \frac{\partial^2 f}{\partial x^2} - 2r^2 \sin \theta \frac{\partial^2 f}{\partial x \partial y} + r^2 \cos^2 \theta \frac{\partial^2 f}{\partial y^2} - r \cos \theta \frac{\partial f}{\partial x} - r \sin \theta \frac{\partial f}{\partial y}$$

$$19 \quad \frac{\partial^2 z}{\partial x^2} = a^2 f''(u), \quad \frac{\partial^2 z}{\partial y^2} = b^2 f''(u), \quad \frac{\partial^2 z}{\partial x \partial y} = ab f''(u) \quad 21 \quad \frac{\partial^2 z}{\partial x^2} = g''(x), \quad \frac{\partial^2 z}{\partial y^2} = h''(y), \quad \frac{\partial^2 z}{\partial x \partial y} = 0$$

$$23 \quad \frac{\partial^2 z}{\partial x^2} = n(n-1)u^{n-2} \left( \frac{\partial u}{\partial x} \right)^2 + nu^{n-1} \frac{\partial^2 u}{\partial x^2}, \quad \frac{\partial^2 z}{\partial y^2} = n(n-1)u^{n-2} \left( \frac{\partial u}{\partial y} \right)^2 + nu^{n-1} \frac{\partial^2 u}{\partial y^2},$$

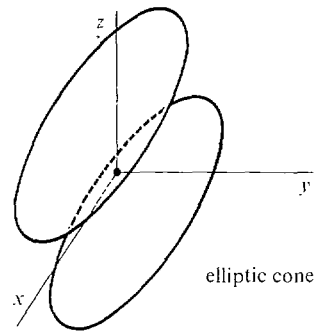
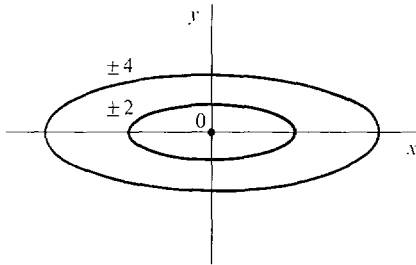
$$\frac{\partial^2 z}{\partial x \partial y} = n(n-1)u^{n-2} \left( \frac{\partial u}{\partial x} \right) \left( \frac{\partial u}{\partial y} \right) + nu^{n-1} \frac{\partial^2 u}{\partial x \partial y}$$

$$25 \quad \frac{\partial^2 z}{\partial s^2} = a \frac{\partial^2 x}{\partial s^2} + b \frac{\partial^2 y}{\partial s^2}, \quad \frac{\partial^2 z}{\partial t^2} = a \frac{\partial^2 x}{\partial t^2} + b \frac{\partial^2 y}{\partial t^2}, \quad \frac{\partial^2 z}{\partial s \partial t} = a \frac{\partial^2 x}{\partial s \partial t} + b \frac{\partial^2 y}{\partial s \partial t}$$

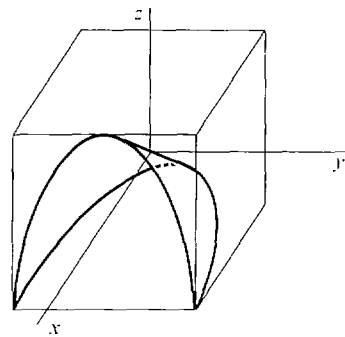
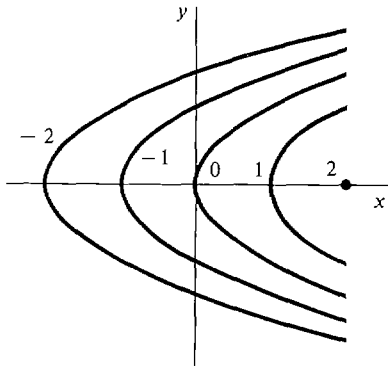
$$27 \quad \frac{\partial^2 z}{\partial s^2} = f'(x)[(g'(s))^2] + g'(s)g''(s), \quad \frac{\partial^2 z}{\partial t^2} = f'(x)[(h'(t))^2] + h'(t)h''(t), \quad \frac{\partial^2 z}{\partial s \partial t} = f'(x)g'(s)h'(t)$$

Extra Problems for Chapter 11

1



3



- 5  $x \neq 0$     7  $1/x + 1/y > 0$     9  $f_x(x, y) = a, f_y(x, y) = b$   
 11  $\frac{\partial z}{\partial x} = \frac{1}{x \ln y}, \frac{\partial z}{\partial y} = -\frac{\ln x}{y(\ln y)^2}$     13  $\Delta z = -\frac{\Delta x}{x(x + \Delta x)} - \frac{2 \Delta y}{y(y + \Delta y)}, dz = -\frac{dx}{x^2} - \frac{2 dy}{y^2}$   
 15  $-8y + z = 4$     17  $\frac{dz}{dt} = -\frac{2 \log_{(2t+1)}(3t+2)}{(2t+1) \ln(2t+1)} + \frac{3}{(3t+2) \ln(2t+1)}$     19 1  
 21  $y = -8x + 17$ , slope =  $-8$     23 max = 4 at (0, 0) and (3, 3); min =  $-1$  at (1, 2)  
 25 no max; no min    27  $\frac{\partial^2 z}{\partial x^2} = -\frac{2xy^3}{(1+x^2y^2)^2}, \frac{\partial^2 z}{\partial y^2} = -\frac{2x^3y}{(1+x^2y^2)^2}, \frac{\partial^2 z}{\partial x \partial y} = -\frac{1-x^2y^2}{(1+x^2y^2)^2}$   
 29  $\frac{\partial^2 z}{\partial t^2} = \cosh^2 \theta \frac{\partial^2 f}{\partial x^2} + 2 \sinh \theta \cosh \theta \frac{\partial^2 f}{\partial x \partial y} + \sinh^2 \theta \frac{\partial^2 f}{\partial y^2}$

Section 12.1

- 1  $168/64 = 2.625$     3 44    5 1.025    7  $(\pi/6)^2(7 + 4\sqrt{3}) \sim 3.8$   
 9  $e^{-5} + e^{-4} + 2e^{-3} + 2e^{-2} + 2e^{-1} + 2 + 2e + 2e^2 + e^3 + e^4 \sim 97.52$   
 11  $150/64 \sim 2.34$     13  $((11 + 7\sqrt{2})\pi)/32 \sim 2.05$     15 108

Section 12.2

- 1  $7/2 = 3.5$     3  $128/3 = 42\frac{2}{3}$     5  $\frac{3}{2} \ln 2 \sim 1.04$     7 4  
 9  $\frac{1}{2}(e^6 - e^2 - e^{-2} + e^{-6}) \sim 197.9$     11  $13/6 \sim 2.17$     13  $32/15 \sim 2.13$   
 15  $36\pi \sim 113.1$     17  $14/3$     19 30    21 2    23  $\frac{4}{15}(212 - 36\sqrt{6})$     25  $e - 1$   
 27  $8/3$     29  $3^8/40$   
 31  $0 \leq x \leq 5, 0 \leq y \leq 5 - x$ , or  $0 \leq y \leq 5, 0 \leq x \leq 5 - y; \int_0^5 \int_0^{5-x} f(x, y) dy dx$ , or  $\int_0^5 \int_0^{5-y} f(x, y) dx dy$

- 33  $-2 \leq x \leq 2, -\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2}, \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x, y) dy dx$   
 35  $-4 \leq x \leq 1, 3x \leq y \leq 4-x^2, \int_{-4}^1 \int_{3x}^{4-x^2} f(x, y) dy dx$   
 37  $-1 \leq y \leq 1, 1/2 \leq x \leq 1/(1+y^2), \int_{-1}^1 \int_{1/2}^{1/(1+y^2)} f(x, y) dx dy$     39  $\pi/4$     41  $32/3$   
 43  $\frac{1}{2}\pi ab$

**Section 12.3**

- 1  $3/4$     3 1    5  $4/3$     7  $16/15$     9  $8/3$     11  $8\pi$     13  $8/15$     15 144  
 17  $\frac{4}{3}\pi abc$

**Section 12.4**

- 1 (a)  $4abk$  (b)  $(0, 0)$  (c)  $\frac{4}{3}(ba^3 + ab^3)k$     3 (a)  $\frac{2}{3}k$  (b)  $(\frac{3}{8}, \frac{3}{8})$  (c)  $\frac{44k}{105}$   
 5 (a)  $26/3$  (b)  $(19/13, 151/65)$  (c)  $944/15$   
 7 (a)  $\frac{2}{7} + \frac{1}{6}$  (b)  $(\frac{2}{7} + \frac{2}{15}, \frac{1}{7} + \frac{1}{15})$  (c)  $\frac{2}{11} + \frac{1}{9} + \frac{2}{45} + \frac{1}{28}$   
 9 (a)  $e^2 + e^{-2} - 2$  (b)  $(\frac{e^2 + 3e^{-2} - 2}{e^2 + e^{-2} - 2}, \frac{e^4 + e^{-4} - 2}{4(e^2 + e^{-2} - 2)})$   
 (c)  $\frac{1}{9}e^6 + 2e^2 - \frac{2}{9} - 10e^{-2} + \frac{1}{9}e^{-6}$     11 2    13 (a)  $(\sqrt{2}/48)g$  (b)  $g/24$   
 15  $m = \frac{8}{3}, W = \frac{16}{15}g$

**Section 12.5**

- 1  $6\pi$     3  $8\pi$     5  $\frac{81}{2}\pi$     7  $2\pi(1 - \ln 2)$     9  $4/21$     11  $\pi/48$   
 13  $32/9$     15  $8\pi^8 + \frac{64}{15}\pi^5$     17  $5\pi/3$     19  $\pi/3 - 4/9$     21  $\frac{1}{3}(\beta - \alpha)(b^3 - a^3)$   
 23 (a)  $\pi b^3/3$  (b)  $\pi b^5/10$     25  $(\pi k/2)(b^4 - a^4)$     27 (a)  $\bar{x} = 4/(3\pi), \bar{y} = 4/(3\pi)$  (b)  $\pi k/8$   
 29 (a)  $\bar{x} = 0, \bar{y} = \frac{3\sqrt{3} + 8\pi}{6\sqrt{3} + 4\pi}b$  (b)  $(\frac{5}{6}\pi + \frac{7\sqrt{3}}{8})b^4k$     31 (a)  $\pi$  (c)  $\sqrt{\pi}$

**Section 12.6**

- 1 12    3 12    5  $13/56$     7  $16/27$     9 40  
 11  $\frac{763}{594} \times 10^9 = (\frac{4}{27} + \frac{100}{88}) \times 10^9 \sim 1.3 \times 10^9$   
 13  $\frac{e^6}{24} - \frac{e^4}{15} + \frac{e^{-2}}{24} - \frac{e^{-6}}{60}$     15  $\frac{52}{63} - \frac{12\sqrt{3}}{35}$   
 17 (a) 3 (b)  $(\frac{19}{36}, \frac{5}{9}, \frac{7}{12})$  (c)  $I_z = \frac{9}{4}, I_y = \frac{7}{3}, I_x = \frac{29}{12}$   
 19 (a) 1 (b)  $(\frac{3}{4}, \frac{5}{12}, \frac{7}{12})$  (c)  $I_z = \frac{5}{6}, I_y = \frac{11}{10}, I_x = \frac{11}{15}$   
 21 (a)  $\frac{5}{18}$  (b)  $(\frac{3}{20}, \frac{3}{20}, \frac{65}{972})$  (c)  $I_z = \frac{5}{16}, I_y = \frac{73}{960}, I_x = \frac{73}{960}$   
 23 (a)  $\frac{1}{6}abck$  (b)  $(\frac{a}{4}, \frac{b}{4}, \frac{c}{4})$  (c)  $I_z = \frac{abck}{60}(a^2 + b^2), I_y = \frac{abck}{60}(a^2 + c^2), I_x = \frac{abck}{60}(b^2 + c^2)$   
 25 (a)  $abck$  (b)  $(\frac{a}{2}, \frac{b}{2}, \frac{c}{2})$  (c)  $I_z = \frac{abck}{3}(a^2 + b^2), I_y = \frac{abck}{3}(a^2 + c^2), I_x = \frac{abck}{3}(b^2 + c^2)$

**Section 12.7**

- 1  $\frac{4\pi}{3}$     3  $\frac{\pi}{10}$     5  $\frac{128}{7}$     7  $\frac{2\pi hb^3}{3}$     9  $\frac{\pi hb^4}{10}$     11  $(0, 0, \frac{2}{3})$   
 13  $\frac{b^4 ck\pi}{2} + \frac{2b^2 c^3 k\pi}{3}$     15  $\frac{\pi hb^4 k}{10}$     17  $\frac{4\pi b^5}{5}$     19  $\frac{4\pi}{15}$     21  $\frac{4\pi b^4}{3}$   
 23  $\frac{\pi b^5}{10}(1 - \cos(2\alpha))$  or  $\frac{\pi b^5}{5}\sin^2 \alpha$     25  $\frac{4}{3}\pi(b^3 - a^3)$     27  $\frac{\pi b^3}{6}(1 - \cos^4 \beta)$     29  $\pi c^4$   
 31  $\frac{8\pi kb^5}{15}$     33  $b^3(\frac{2\pi}{3} - \frac{8}{9})$     35  $\frac{3}{8}b$  from the center

## Extra Problems for Chapter 12

- 1 0 3 0 5  $\frac{1}{3}$  7  $\frac{4}{5}$  9  $\frac{1}{2}(\ln 2)^2 - \frac{1}{6}(\ln 2)^3$  11  $\frac{1}{6}$   
 13  $m = 2k, (\bar{x}, \bar{y}) = (\pi/2, \pi/8), I_0 = (\pi^2 - \frac{32}{9})k$  15  $m = \pi r^4/4, (\bar{x}, \bar{y}) = (0, 0), I_0 = \pi r^6/6$   
 17  $45\pi$  19  $2\pi a$  23  $311/3960$   
 25  $(\bar{x}, \bar{y}, \bar{z}) = (\frac{1}{5}, \frac{1}{5}, \frac{1}{30}), I_z = \frac{k}{60}, I_y = I_x = \frac{29k}{3360}$   
 27  $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, \frac{b^2 + c}{2}), I_z = \frac{\pi b^4 c k}{2}$  29  $m = \pi(b^4 - a^4), I = \frac{4\pi}{9}(b^6 - a^6)$

## Section 13.1

- 1  $\text{grad } f = 2xi + 2yj, f_U = \sqrt{2}x + \sqrt{2}y$  3  $\text{grad } f = 2xy^3i + 3x^2y^2j, f_U = \frac{6}{5}xy^3 - \frac{1}{5}x^2y^2$   
 5  $\text{grad } f = -\sin x \sin y i + \cos x \cos y j, f_U = (1/\sqrt{5})(-\sin x \sin y + 2 \cos x \cos y)$   
 7  $\text{grad } f = \frac{x}{\sqrt{x^2 + y^2}}i + \frac{y}{\sqrt{x^2 + y^2}}j, f_U = \frac{x - y}{\sqrt{2x^2 + 2y^2}}$   
 9  $\text{grad } f = yzi + xzj + xyk, f_U = \frac{1}{3}(yz + 2xz - 2xy)$   
 11  $\text{grad } f = -x^{-2}i - 2y^{-2}j - 3z^{-2}k, f_U = (1/\sqrt{2})(-x^{-2} + 3z^{-2})$   
 13  $\text{grad } f = \frac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}}, f_U = \frac{x \cos \alpha + y \cos \beta + z \cos \gamma}{\sqrt{x^2 + y^2 + z^2}}$  15  $7\sqrt{10}/20$   
 17  $9a/\sqrt{a^2 + b^2}$  19  $-1/\sqrt{3}$  21  $(1/\sqrt{2})(i + j)$  23  $\frac{1}{2}(3i + 6j + 2k)$   
 25  $(f_U)_V = \frac{\partial^2 z}{\partial x^2} u_1 v_1 + \frac{\partial^2 z}{\partial x \partial y} (u_1 v_2 + u_2 v_1) + \frac{\partial^2 z}{\partial y^2} u_2 v_2$

## Section 13.2

- 1  $2e^3$  3  $\frac{9}{2}e - \frac{3}{2}$  5  $\sin 1 - \cos 1 + 1$  7  $\frac{3}{20} + \frac{5}{2} \ln 4$  9  $-2\pi, 2\pi$   
 11  $-4\pi$  13  $\frac{11}{30}$  15  $\frac{1}{2}$  17  $\frac{1}{2} \ln(\frac{29}{2})$  19  $1243/3$

## Section 13.3

- 1 no potential function 3 no potential function 5 no potential function  
 7  $y \sin x + C$  9  $-2x + 6y + C$  11 no potential function  
 13  $\frac{3}{2}x^2 + 5xy - y^2 + C$  15  $\cosh x \cosh y + C$  23  $\frac{3}{2}x^2 + 4xy - y^2 = C$   
 25  $\frac{2}{3}x^{3/2} + x\sqrt{y} = C$  27  $\frac{1}{3}y^3 + y \arctan x = C$  29  $\cos x \sin y = C$   
 31  $(x^2/2) + (y^2/2) + (2/3)(x + y)^{3/2} = C$  33  $P(x, y) = 2 \sin x \cos x \sin y$

## Section 13.4

- 1 1 3  $\frac{1}{2}(-e^7 + e^{-1} + e - e^{-7})$  5  $-\frac{1}{6}$  7  $\frac{1}{2} - \ln 2$  9  $-17/12$   
 11  $-\pi/2$  13 (a)  $-x - y$  (b)  $-1$  (c)  $y - x$  (d) 0  
 15 (a)  $2bx - 2ay$  (b)  $(2b - a)/3$  (c) 0 (d) 0 17 (a)  $-2$  (b)  $-2\pi$  (c) 0 (d) 0  
 19  $\pi a^2 + \pi/2$

## Section 13.5

- 1  $25\sqrt{21}/4$  3  $(\pi/6)(5\sqrt{5} - 1)$  5  $\pi a^2/2$  7  $(\pi/6)[(1 + 4a^2)^{3/2} - 1]$   
 9  $a^2(\pi - 2)$  11  $A\sqrt{1 + a^2 + b^2}$  13  $8a^2$  15 16 17  $-\pi$  19  $-2e^{-1}$

## Section 13.6

- 1  $\text{curl } F = 0, \text{div } F = 2x + 2y + 2z$  3  $\text{curl } F = -i + j - k, \text{div } F = 3$   
 5  $\text{curl } F = (ze^{x+y} - ye^{x+z})i + (xe^{y+z} - ze^{x+y})j + (ye^{x+z} - xe^{y+z})k,$   
 $\text{div } F = e^{y+z} + e^{x+z} + e^{x+y}$  9 0 11  $A(p(c - b) + q(a - c) + (a - b))$   
 13  $abc(a + b + c)$  15 3 17  $\pi/3$  19  $\pi/2$

## Extra Problems for Chapter 13

- 1  $-\sin x \cos \alpha + \cos y \sin \alpha$     3  $\text{grad } f = ye^{xy}\mathbf{i} + xe^{xy}\mathbf{j}, f_u = e^{xy}(y \cos \alpha + x \sin \alpha)$   
 5  $\frac{1}{\sqrt{14}}(-2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$     7  $\ln\left(\frac{6\sqrt{2}}{5}\right)$     9 0    11  $x \ln x + xy \ln y - x$   
 13  $\frac{3}{2}y^2 - e^{-y} \cos x = C$     15  $\frac{131}{60}$     17  $\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{6}$     19  $2\pi$   
 21  $\text{curl } \mathbf{F} = (x^2z - 3xy^2z^2)\mathbf{i} + (xy - 2xyz)\mathbf{j} + (y^2z^3 - xz)\mathbf{k}, \text{div } \mathbf{F} = yz + 2xyz^3 + x^2y$

## Section 14.1

- 1  $y = -1/2 \cos(t^2) + C$     3  $y = \ln(t + C)$     5  $y = \frac{1 + Ce^{2t}}{1 - Ce^{2t}}, y = 1, y = -1$   
 7  $y = -\frac{1}{0.5t^2 + C}, y = 0$     9  $y = \tan(e^t + C)$     11  $y = C \sec t, y = 0$   
 13  $y = \left(\frac{t^2}{4} + 9\right)^{1/2}$     15  $y = -\sqrt{2(t \ln t - t) + 6}$     17  $y = 2$

## Section 14.2

- 1  $y = C \cdot e^{-5t}$     3  $y = C \cdot e^{\arctan t}$     5  $y = 4 \cdot e^{-t}$     7  $y = e \cdot e^{\cos t}$     9  $y = 0$   
 11  $y = 4t^2$     13  $y = e^{1-t^2}$     15  $y = 100 \cdot e^{-t \cdot \ln 5}$     17  $y = 1,000,000 \cdot e^{t \cdot \ln 1.5}$

## Section 14.3

- 1  $y = 2 + Ce^{-4t}$     3  $y = 5 + Ce^{-(1/2)t^2}$     5  $y = -(t^2 + 2t + 2) \cdot Ce^{-4t}$   
 7  $y = \frac{1}{3}t^{-1} + Ct^2$     9  $y = \sin t + C \cos t$     11  $y = e^{-\cos t} \left( \int_0^t s e^{\cos s} ds + C \right)$   
 13  $y = e^{-\int_0^t \cos(e^r) dr} \left[ \int_0^t e^{\int_0^r \cos(e^s) ds} dr + C \right]$   
 15  $-[400,000t + 15,840,000] + 15,940,000e^{0.025t}$   
 17 15,000e - 25,000 dollars, or \$15,774.23

## Section 14.4

	$t$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	$Y(t)$	1.0	1.0	1.01	1.030	1.059	1.097	1.142	1.195	1.253	1.317	1.386
3	$Y(t)$	1.0	1.054	1.095	1.122	1.137	1.140	1.133	1.117	1.093	1.061	1.023

5 Apply the lemma with  $M = 4$ .    7 Apply the lemma with  $M = \pi/2$ .  
 9  $y(t) = 0$  for  $0 \leq t \leq b$ ,  $y(t) = [(2/3)(t - b)]^{3/2}$  for  $b < t < \infty$ .

## Section 14.5

- 1  $10 - i2$     3  $i10$     5  $0.1 - i(0.7)$     7 4    9  $\pm i5$     11  $-1 \pm i2$   
 13  $5 \text{cis}(\pi/2)$     15  $3\sqrt{2} \text{cis}(-3\pi/4)$     17  $2 \text{cis}(-\pi/6)$     19  $-i2/3$   
 21  $2 \text{cis}(-\pi/12)$     23  $-4 + i4$     25  $\pm 2^{1/4} \text{cis}(\pi/8)$     27  $\pm 2 \text{cis}(-\pi/4)$ ,  
 or  $\pm \sqrt{2}(1 - i)$     29  $ie^{-3}$     31  $e\sqrt{2}/2 - ie\sqrt{2}/2$     33  $(5 - i3)e^{(5-i3)t}$   
 35  $(3 + i2)e^{(2-i7)+(3+i2)t}$     37  $Ce^{(-2+i3)t}$     39  $Ce^{(3-i5)t}$   
 41  $e^{i+(2-i)t}$ , or  $e^{2t} \text{cis}(1 - t)$     43  $4e^{(2-i)t}$ , or  $4e^{2t} \text{cis}(-t)$

**Section 14.6**

- 1  $Ae^{2t} + Be^{-3t}$     3  $Ae^{5t} + Bte^{5t}$     5  $A \cos(4t) + B \sin(4t)$   
 7  $e^t[A \cos t + B \sin t]$     9  $-\frac{1}{4}e^{-5t} + \frac{5}{4}e^{-t}$     11  $5e^{-6t} + 20te^{-6t}$   
 13  $-2 \cos(\sqrt{5}t) + \sqrt{5} \sin(\sqrt{5}t)$     15  $e^{-6t}[4 \cos t + 24 \sin t]$   
 17  $2 \cos(2t - \pi/6)$ , amplitude = 2, frequency = 2, phase shift =  $\pi/6$   
 19  $e^{-2t}\sqrt{2} \cos(3t - \pi/4)$ , amplitude =  $\sqrt{2}e^{-2t}$ , frequency = 3, phase shift =  $\pi/4$   
 21  $e^{-2t}[2 \cos(5t) + \sin(5t)]$

**Section 14.7**

- 1  $(24/676) \cos t - (10/676) \sin t$     3  $(1/2)t^2 + (3/16)t - (3/8)$     5  $0.5e^{2t}$   
 7  $-0.2te^{-3t}$     9  $0.125t \sin(4t)$     11  $3t^2e^{-6t}$     13  $Ae^{-3t} + Be^{-2t} + 0.8$   
 15  $A \cos(\sqrt{5}t) + B \sin(\sqrt{5}t) + 2 \sin(2t)$     17  $4e^t + e^{-t} - 3t - 5$   
 19  $e^{-6t}[2 \cos t + 20 \sin t] + 2e^{-4t}$   
 21  $e^{-2t}[A \cos(5t) + B \sin(5t)] + 25 \cos(2t) + 8 \sin(2t)$ , steady state =  $25 \cos(2t) + 8 \sin(2t)$   
 25 5

**Extra Problems for Chapter 14**

- 1  $y = -1/(\sin t + C)$     3  $y = -\ln(-\ln|t| + e^{-2})$     5  $y = Ce^{-5t^2}$     7  $y = e^{-6t}$   
 9  $y = (2/3)t - 2/9 + Ce^{-3t}$     11  $y = 3,500,000e^{0.02t} - 2,500,000$   
 15  $y = Ae^{4t} + Be^t$     17  $y = e^{2t}[A \cos(2t) + B \sin(2t)]$     19  $y = -(1/6)e^{-5t} + (1/6)e^t$   
 21  $y = e^{-(3/2)t}[10 \cos(t/2) + 30 \sin(t/2)]$     23  $y = Ae^{4t} + Be^t + 13/16 + t/4$   
 25  $y = -2 \cos t - 3 \sin t - (1/6)e^{-5t} + (13/6)e^t$



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