Mise en pratique for the definition of the kilogram in the SI

Consultative Committee for Mass and Related Quantities

1. Introduction

The purpose of this *mise en pratique*, prepared by the Consultative Committee for Mass and Related Quantities (CCM) of the International Committee for Weights and Measures (CIPM), is to indicate how the definition of the SI base unit, the kilogram, symbol kg, may be realized in practice.

In general, the term "to realize a unit" is interpreted to mean the establishment of the value and associated uncertainty of a quantity of the same kind as the unit that is consistent with the definition of the unit. The definition of the kilogram does not imply any particular experiment for its practical realization. Any method capable of deriving a mass value traceable to the set of seven reference constants could, in principle, be used. Thus, the list of methods given is not meant to be an exhaustive list of all possibilities, but rather a list of those methods that are easiest to implement and/or that provide the smallest uncertainties and which are officially recognized as primary methods by the relevant Consultative Committee.

A primary method is a method having the highest metrological properties; whose operation can be completely described and understood; for which a complete uncertainty statement can be written down in terms of SI units; and which does not require a reference standard of the same quantity.

2. Definition of the kilogram

2.1. Definition

The definition of the kilogram, SI base unit of mass, is as follows [2.1]:

> **The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be 6.626 070 15 × 10^{-34} when expressed in the unit J s, which is equal to kg m^2 s^{-1}, where the metre and the second are defined in terms of c and $\Delta\nu_{Cs}$.**

Thus the Planck constant h is exactly $h = 6.626\,070\,15 \times 10^{-34}$ J s. This numerical value of h defines the unit joule second in the SI and, in combination with the SI second and metre, defines the kilogram. The second and metre are themselves defined by exact values of the hyperfine transition frequency $\Delta\nu_{Cs}$ of the caesium 133 atom and the speed of light in vacuum c. The numerical value of h given in the definition of the kilogram has ensured the continuity of the unit of mass with the previous definition, as explained in section 5.

Details of the redefinition process are described in [2.2].

2.2. Traceability chain for mass metrology

The definition of the unit of mass does not imply or suggest any particular method to realize it. This document recommends primary methods of practical realization of the mass unit based on its formal definition. A primary method is a method for determining a mass in terms of h without use of any other mass standard (Figure 1). The mass whose value is to be determined may be an artefact, atom or other entity although the following focuses on metrology for mass artefacts at the highest level of accuracy. Such an artefact whose mass has been directly calibrated by a primary method to realize the kilogram definition is called a primary mass standard in this document. Secondary mass standards are established through calibration with respect to primary mass standards.

The current primary methods focus on the realization and dissemination of the unit of mass at a nominal value of 1 kg. The *mise en pratique* may be updated to include information on primary methods at different nominal mass values.

Primary methods for the realization of the definition of the kilogram and procedures for its dissemination through primary mass standards are described in the following two sections. The traceability chain is shown schematically in Figure 1.

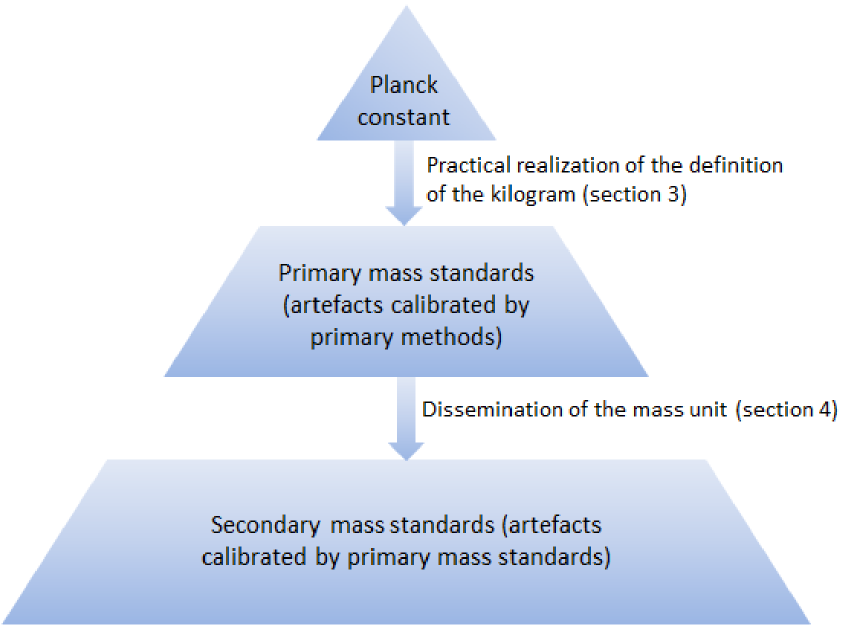

Figure 1. Illustration of the traceability chain from the definition of the kilogram. The unit of the Planck constant being kg m^2 s^{-1}, the units second and metre are needed to derive a primary mass standard from the Planck constant.

This *mise en pratique* will be updated to take account of new methods and technological improvements. It is not printed in the *SI Brochure* [2.1], but the current version is posted on the open BIPM web site at https://www.bipm.org/en/publications/si-brochure/.

3. Practical realization of the definition of the kilogram

There are currently two independent primary methods that are capable of realizing the definition of the kilogram with relative uncertainties within a few parts in 10^8. The first of these relies on determining the unknown mass using an electromechanical balance specially designed for the purpose. The second method compares the unknown mass to the mass of a single atom of a specified isotope by counting the number of atoms in a crystal, where the mass of the atom is well-known in terms of h, c and $\Delta\nu_{Cs}$.

3.1. Realization by comparing electrical power to mechanical power

Accurate instruments that function in a way that electrical and mechanical power can be equated had been known as watt balances, and, more recently, as Kibble balances[1]. Kibble balances can be designed with different geometries and operated with different experimental protocols. The following schematic description serves to demonstrate that any of these Kibble-balance configurations has the potential to be a primary method to realize the definition of the kilogram.

The determination of the unknown mass m_x of an artefact x is carried out in two modes: the weighing mode and the moving mode. They may occur successively or simultaneously. In the weighing mode, the weight[2] $m_x\, g$ of the artefact is balanced by the electromagnetic force produced, for example, on a circular coil of wire-length l immersed in a radial magnetic field of flux density B when a current I_1 flows through the coil. The magnet and coil geometries are designed to produce a force that is parallel to the local gravitational acceleration. The acceleration of gravity g acting on the mass, and the current I_1 flowing in the coil are measured simultaneously so that

$$m_x\, g = I_1\, B\, l. \tag{3.1}$$

In the moving mode, the voltage U_2 which is induced across the terminals of the same coil moving vertically at a velocity v through the same magnetic flux density, is measured so that

$$U_2 = v\, B\, l. \tag{3.2}$$

The equations describing the two modes are combined by eliminating Bl:

$$m_x\, g\, v = I_1\, U_2 \quad . \tag{3.3}$$

Thus power of a mechanical nature is equated to power of an electromagnetic nature. The powers are manifestly "virtual" in this method of operation because power does not figure in either mode of this two-mode experiment.

The current I_1 can, for example, be determined using Ohm's law by measuring the voltage drop U_1 across the terminals of a stable resistor of value R. Both voltages, U_1 and U_2, are measured in terms of the Josephson constant K_J, which is taken to be $K_J = 2e/h$; e is the elementary charge. Similarly, R is measured in terms of the von Klitzing constant R_K, which is taken to be $R_K = h/e^2$. The quantities v and g are measured in their respective SI units, $\mathrm{m\,s^{-1}}$ and $\mathrm{m\,s^{-2}}$. Note that $K_J^2 R_K = 4/h$ allowing (3.3) to be rewritten schematically as

$$m_x = h\left(\frac{bf^2}{4}\right)\frac{1}{g v} \quad , \tag{3.4}$$

[1] We refer to watt balances as "Kibble balances" to recognize Dr. Bryan Kibble, who originally conceived the idea of this experiment.

[2] In legal metrology "weight" can refer to a material object or to a gravitational force. The terms "weight force" and "weight piece" are used in legal metrology if the meaning of "weight" is not clear from the context [3.1].

where f is an experimental frequency and b is a dimensionless experimental quantity, both associated with the required measurements of electrical current and voltage.
See [3.2] for a more complete analysis of this experiment.

All relevant influences on the mass, m_x, as derived from (3.4) must be considered for the realization, maintenance and dissemination of the unit of mass (see also Annex A2).
Other electromagnetic and electrostatic realizations have been proposed, such as the joule-balance and volt-balance methods, and may well be perfected [3.2, 3.3].

3.2. Realization by the X-ray-crystal-density method

The concept of the X-ray-crystal-density (XRCD) method comes from a classical idea where the mass of a pure substance can be expressed in terms of the number of elementary entities in the substance[3]. Such a number can be measured by the XRCD method in which the volumes of the unit cell and of a nearly perfect crystal are determined, e. g. by measuring the lattice parameter a and the mean diameter of a spherical sample. Single crystals of silicon are most often used in this method because large crystals can be obtained having high chemical purity and no dislocations. This is achieved using the crystal growth technologies developed for the semiconductor industry. The macroscopic volume V_s of a crystal is equal to the mean microscopic volume per atom in the unit cell multiplied by the number of atoms in the crystal. For the following, assume that the crystal contains only the isotope ^{28}Si. The number N of atoms in the macroscopic crystal is therefore given by

$$N = 8V_s/a(^{28}\mathrm{Si})^3, \tag{3.5}$$

where 8 is the number of atoms per unit cell of crystalline silicon and $a(^{28}\mathrm{Si})^3$ is the volume of the unit cell, which is a cube; i.e., $V_s/a(^{28}\mathrm{Si})^3$ is the number of unit cells in the crystal and each unit cell contains eight silicon 28 atoms. Since the volume of any solid is a function of temperature and, to a lesser extent, hydrostatic pressure, V_s and $a(^{28}\mathrm{Si})^3$ are referred to the same reference conditions. For practical reasons, the crystal is fashioned into a sphere having a mass of approximately 1 kg.

To realize the definition of the kilogram, the mass m_s of the sphere is first expressed in terms of the mass of a single atom, using the XRCD method[4]:

$$m_s = N\, m(^{28}\mathrm{Si}), \tag{3.6}$$

Since the experimental value of the physical constant $h/m(^{28}\mathrm{Si})$ is known to high accuracy [3.5], one can rewrite (3.6) as

$$m_s = h\, N\left(\frac{m(^{28}\mathrm{Si})}{h}\right). \tag{3.7}$$

The XRCD experiment determines N; $m(^{28}\mathrm{Si})/h$ is a constant of nature whose value is known to

[3] The measurements described here were first used to determine the value of the Avogadro constant N_A, which is defined as the number of elementary entities per mole of substance. An accurate measurement of N_A was an essential contribution on the road to redefining the kilogram in 2018. Today, however, the numerical value of N_A is exactly defined when expressed in the SI unit mol^{-1} thus making the definition of the mole independent of the kilogram.
[4] It is well known that (3.6) is not exact because the right-hand side is reduced by the mass equivalent, E/c^2, of the total binding energy E of the atoms in the crystal, where c is the speed of light in vacuum. The correction, about 2 parts in 10^{10} [3.4], is insignificant compared with present experimental uncertainties and has therefore been ignored. Additional energy terms (e.g. thermal energy) are even smaller than the binding energy and thus negligible.

high accuracy and, of course, the numerical value of h is now fixed.

The sphere is a primary mass standard and the unit of mass, the kilogram, is disseminated from this standard. Spheres currently used in this work are enriched in the isotope ^{28}Si but the presence of trace amounts of two additional silicon isotopes leads to obvious modifications of the simple equations presented in this section. See [3.6] for a more complete analysis of this experiment.
All relevant influences on the mass of the sphere, m_s, as derived from (3.7) must be considered for the realization, maintenance and dissemination of the unit of mass (see also Annex A2).

4. Dissemination of the mass unit

The definition of the kilogram ensures that the unit of mass is constant in time and that the definition can be realized by any laboratory, or collaboration of laboratories, with the means to do so. Any National Metrology Institute (NMI), Designated Institute (DI), the Bureau International des Poids et Mesures (BIPM), or collaboration among them, that realizes the kilogram definition can disseminate the SI kilogram from its primary mass standards to any other laboratory or, more generally, to any user of secondary mass standards (see Figure 1). This is described in section 4.1[5]. Dissemination from a dedicated ensemble of 1 kg secondary standards maintained at the BIPM, called BIPM ensemble of reference mass standards, is described in section 4.2.

4.1. Dissemination from a particular realization of the kilogram

The dissemination of the mass unit is based on primary mass standards obtained from the realization of the definition of the kilogram according to the methods described in section 3. All relevant influences on a primary mass standard must be considered for the maintenance and dissemination of the mass unit (see Annex A2). In particular, the uncertainty due to a possible drift of the primary mass standards since the last realization must be taken into account.

The BIPM in coordination with the CCM organizes a CIPM key comparison [4.5], CCM.M-K8 [4.6], for laboratories with primary realization methods. In this comparison, the primary mass standards of the participants are compared with each other and with stable BIPM mass standards which maintain the reference values of previous comparisons (see section 4.2). The CCM decides the required periodicity of laboratory participation in CCM.M-K8 in order to support relevant calibration and measurement capabilities (CMCs).

In cases where compliance with the CIPM Mutual Recognition Arrangement (CIPM MRA) is required [4.7], it is essential that the mass standards are traceable to primary mass standards of a participant in CCM.M-K8 that has relevant CMC entries or, in the case of the BIPM, suitable entries in its calibration and measurement services as approved by the CIPM. Dissemination of the whole mass scale is validated for all NMIs/DIs and the BIPM through the traditional types of key comparisons organized prior to the present definition of the kilogram.
Results of all key comparisons are published in the Key Comparison Database (KCDB) in accordance with the rules of CIPM MRA [4.5] and may be used in support of NMI/DI claims of its calibration and measurement capabilities and the BIPM claims listed in its calibration and measurement services.

[5] In order to preserve the international equivalence of calibration certificates, the National Metrology Institutes having a realization of the kilogram avail themselves of the consensus value until the dispersion of the results from individual realization experiments is compatible with the uncertainties of the individual realizations [4.1]. The consensus value is obtained from a statistical analysis of all the data from available realizations of the kilogram. The consensus value is managed by a CCM task group to ensure stability and continuity, taking all new realizations and comparisons into account. Its calculation is described in the "CCM detailed note on the dissemination process after the redefinition of the kilogram" [4.2]. See also [4.3], [4.4] and Annex A3, which all address issues related to the dissemination of the kilogram from multiple realizations of its definition.

4.2. Dissemination from the BIPM

In accordance with Resolution 1 of the 24th meeting of the General Conference on Weights and Measures (CGPM) (2011) [4.8] and Resolution 1 of the 25th meeting of the CGPM (2014) [4.9], the BIPM maintains an ensemble of reference mass standards "*to facilitate the dissemination of the unit of mass*" in the revised SI. This ensemble is composed of 1 kg artefacts of various materials which have been chosen to minimize known or suspected sources of mass instability.
The average mass of the ensemble is derived from links to primary realizations of the kilogram definition that have participated in an initial pilot study [4.10] and/or in CCM.M-K8 through an algorithm defined by the CCM. The BIPM[6] disseminates the unit of mass from the average mass of the ensemble. NMIs, DIs, the BIPM or collaborations among them, may adopt a similar strategy for dissemination of the mass unit.

5. Continuity with the previous definition of the kilogram

Preserving the continuity of measurements traceable to an SI unit before and after its redefinition is a generally accepted criterion for revised definitions of SI base units. The previous definition of the kilogram was based on the mass of the international prototype of the kilogram (IPK) immediately after the prescribed cleaning procedure. The dissemination of the mass unit therefore required traceability to the mass of the IPK.

5.1. Steps to ensure continuity

Prior to the adoption of Resolution 1 of the 26th CGPM (2018) [5.1], all mass standards used for the experimental determination of the Planck constant were calibrated by an "extraordinary use" of the IPK [5.2]. Additionally, the BIPM ensemble of reference mass standards was calibrated.

A pilot study was performed in 2016 to prepare for the redefinition of the kilogram [4.10]. The comparison included all available experiments capable of determining the value of the Planck constant to high accuracy.

In preparation for the redefinition of the kilogram (and other units) the Committee on Data for Science and Technology (CODATA) Task Group on Fundamental Constants evaluated all published experimental values for the Planck constant h by July 1st 2017 and recommended the numerical value of h to be used for the new definition of the kilogram [5.3]. The relative uncertainty of h recommended by the Task Group was assigned to the international prototype of the kilogram just after fixing the numerical value of h. As a consequence the 26th CGPM confirmed in its Resolution 1 that, just after the redefinition, the mass of the IPK was still 1 kg, but within an uncertainty of 1.0×10^{-8}. Accordingly, all mass values traceable to the IPK were unchanged when the new definition came into effect, but all associated uncertainties of these mass values were increased by a common component of relative uncertainty, equal to the relative uncertainty of the IPK just after the redefinition.

5.2. The role and status of the international prototype

The mass values of the IPK and its six official copies are now determined experimentally with traceability to realization experiments (see Section 4).

Subsequent changes to the mass of the IPK may have historical interest even though the IPK no longer retains a special status or a dedicated role in this *mise en pratique* [5.4]. By following the

[6] The BIPM operates under a quality management system (QMS) that conforms to ISO/IEC 17025:2005. The QMS is under the exclusive supervision of the CIPM. Competence is demonstrated through on-site audits conducted by external experts and regular reports to CIPM Consultative Committees and Regional Metrology Organizations.

change in mass of the IPK over time, one may be able to ascertain its mass stability with respect to fundamental constants, which has long been a topic of conjecture. For that reason, the IPK and its six official copies are conserved at the BIPM under the same conditions as they were prior to the redefinition.

References

[2.1] BIPM, The International System of Units (SI Brochure) [9th edition, 2019], https://www.bipm.org/en/publications/si-brochure/.

[2.2] Richard P, Fang H and Davis R, "Foundation for the redefinition of the kilogram", *Metrologia* **53** (2016) A6–A11.

[3.1] *OIML D28* (2004), https://www.oiml.org/en/files/pdf_d/d028-e04.pdf

[3.2] Robinson I A and Schlamminger S, "The watt or Kibble balance: a technique for implementing the new SI definition of the unit of mass", *Metrologia* **53** (2016) A46–A74.

[3.3] Shaw G A, Stirling J, Kramar J A, Moses A, Abbott P, Steiner R, Koffman A, Pratt J R and Kubarych Z J, "Milligram mass metrology using an electrostatic force balance", *Metrologia* **53** (2016) A86–A94.

[3.4] Davis R S and Milton M J T, "The assumption of the conservation of mass and its implications for present and future definitions of the kilogram and the mole", *Metrologia* **51** (2014) 169–173.

[3.5] Cladé P, Biraben F, Julien L, Nez F and Guellati-Khelifa S, "Precise determination of the ratio h/m_u: a way to link microscopic mass to the new kilogram", *Metrologia* **53** (2016) A75–A82.

[3.6] Fujii K, Bettin H, Becker P, Massa E, Rienitz O, Pramann A, Nicolaus A, Kuramoto N, Busch I and Borys M, "Realization of the kilogram by the XRCD method", *Metrologia* **53** (2016) A19-A45.

[4.1] CCM Recommendation G 1 (2017), https://www.bipm.org/documents/20126/28430100/working-document-ID-9058/d49e6b1b-1bf8-08cf-ec8c-016ab66ddd9e

[4.2] CCM detailed note on the dissemination process after the redefinition of the kilogram, https://www.bipm.org/documents/20126/28432674/working-document-ID-11291/cf8f685d-fc3d-1883-9a42-0678a2c34453

[4.3] Stock M, Davidson S, Fang H, Milton M, de Mirandés E, Richard P and Sutton C, "Maintaining and disseminating the kilogram following its redefinition", *Metrologia* **54** (2017) S99-S107.

[4.4] Nielsen L, "Disseminating the unit of mass from multiple primary realisations", *Metrologia* **53** (2016) 1306-1316.

[4.5] *Measurement comparisons in the CIPM MRA*, CIPM-MRA-G-11, https://www.bipm.org/en/cipm-mra/cipm-mra-documents

[4.6] Comparison of Realizations of the Kilogram CCM.M-K8, https://www.bipm.org/kcdb/comparison?id=1341

[4.7] *Overview and implementation of the CIPM MRA*, CIPM-MRA-P-11, https://www.bipm.org/en/cipm-mra/cipm-mra-documents

[4.8] Resolution 1 of the 24th CGPM (2011), https://www.bipm.org/en/committees/cg/cgpm/24-2011/resolution-1

[4.9] Resolution 1 of the 25th CGPM (2014), https://www.bipm.org/en/committees/cg/cgpm/25-2014/resolution-1

[4.10] Report on CCM Pilot Study CCM.R-kg-P1, https://www.bipm.org/en/committees/cc/ccm/pilot-studies

[5.1] Resolution 1 of the 26th CGPM (2018), https://www.bipm.org/en/committees/cg/cgpm/26-2018/resolution-1

[5.2] Stock M, Barat P, Davis R S, Picard A and Milton M J T, "Calibration campaign against the international prototype of the kilogram in anticipation of the redefinition of the kilogram part I: comparison of the international prototype with its official copies",

Metrologia **52** (2015) 310–316.
de Mirandés E, Barat P, Stock, M and Milton M J T, "Calibration campaign against the international prototype of the kilogram in anticipation of the redefinition of the kilogram, part II: evolution of the BIPM as-maintained mass unit from the 3rd periodic verification to 2014" *Metrologia* **53** (2016) 1204–1214.
[5.3] Mohr P J, Newell D B, Taylor B N and Tiesinga E., "Data and analysis for the CODATA 2017 Special Fundamental Constants Adjustment," *Metrologia* **55** (2018) 125-146.
[5.4] Davis, R S, "The role of the international prototype of the kilogram after redefinition of the International System of Units", *Phil. Trans. R. Soc. A*, **369** (2011) 3975-3992.

[A1.1] *Mise en pratique* of the definition of the ampere.
[A1.2] *Mise en pratique* of the definition of the kelvin.
[A1.3] BIPM, The International System of Units (SI Brochure) [8th edition, 2006], https://www.bipm.org/en/publications/si-brochure/.

ANNEXES

A1. Traceability to units derived from the kilogram

A1.1. Coherent derived units expressed in terms of base units kg m^p s^q

Neither the realizations of the metre nor the second have been affected by the Resolution 1 of the 26th CGPM. This means that for any coherent derived units expressed in terms of base units as kg m^p s^q (where p and q are integers), the only change in traceability to the SI is in the traceability to the kilogram, and this has been described above. Examples of quantities and their associated coherent derived units are shown in Table A1.1. Several of the coherent derived units have special names, e.g. newton, joule, pascal. These are not given in Table A1.1 but they are tabulated in Table 4 of the 9th edition of the *SI Brochure* [2.1].

Table A1.1. Some quantities whose SI coherent unit is expressed as kg m^p s^q.

Quantity	p	q
mass density	-3	0
surface density	-2	0
pressure, stress	-1	-2
momentum	1	-1
force	1	-2
angular momentum	2	-1
energy, work, torque	2	-2
power	2	-3

A1.2. Electrical units

The ampere was previously defined in terms of the second, the metre and the kilogram, and by giving a fixed numerical value to the magnetic constant μ_0, whose unit is kg m s^{-2} A^{-2} (equivalently, N A^{-2} or H m^{-1}). The ampere is now defined in terms of the second and a fixed numerical value for the elementary charge e, whose unit is A s. The fact that the Planck constant now has a defined numerical value is of great utility to electrical metrology, as described in the *mise en pratique* for the ampere [A1.1].

A1.3. Units involving the kelvin and the candela

The kelvin is now defined in terms of exact numerical values for $\Delta\nu_{\text{Cs}}$, h, and the Boltzmann constant k. The unit of k is kg m^2 s^{-2} K^{-1} (equivalently, J K^{-1}). The redefinition of the kilogram has no practical impact on this change (see the *mise en pratique* of the definition of the kelvin [A1.2]). Similarly, although the definition of the candela refers in part to power, Resolution 1 has had no practical impact on the realization of the candela.

A1.4. Atomic, subatomic and molecular units

(Note: This section focuses on atomic physics rather than chemistry.)

The fact that adoption of Resolution 1 by the 26th CGPM (2018) redefined both the kilogram and the mole, and that the unit of molar mass is kg mol^{-1}, is a potential source of confusion regarding non-SI units such as the unified atomic mass unit, u, commonly used in atomic, subatomic and molecular science. The following describes the present situation and contrasts it with the situation described in the 8th edition of the *SI Brochure* [A1.3]. In Section A1.4.1 we list important equations used in atomic and molecular physics and define the quantities that appear in these equations. Of course the changes to the SI have no effect on the equations. However, uncertainties of the quantities appearing in the equations are affected by the redefinitions of the kilogram and mole. Section A1.4.2 describes these changes and gives present uncertainties.

A1.4.1. Equations of physics

The equations of physics have not changed. Some of the principal relations used in atomic physics are recalled in this subsection.

The unified atomic mass constant m_u is defined in terms of the mass of the ^{12}C isotope

$$m_u = m(^{12}C)/12 \;. \tag{A1.1}$$

The unified atomic mass unit, u, also known as the dalton (symbol: Da), is not an SI unit. Formally, the conversion between u and kg is u = $\{m_u\}$ kg where the curly brackets around m_u mean "the numerical value of m_u when it is expressed in the unit kg".

The relative atomic mass of an elementary entity X is a pure number defined by

$$A_r(X) = m(X)/m_u = 12\, m(X)/m(^{12}C) \tag{A1.2}$$

where $A_r(X)$ is the relative atomic mass of X, and $m(X)$ is the atomic mass of X. (Relative atomic mass is usually called "atomic weight" in the field of chemistry.) The elementary entity X must be specified in each case. If X represents an atomic species, or nuclide, then the notation ^{A}X is used for a neutral atom where A is the number of nucleons; for example: ^{12}C.

In the SI, m_u is determined experimentally in terms of the definition of the kilogram. See the next section for additional information.

The molar mass of X, $M(X)$, is defined as the atomic mass of the entity X multiplied by the Avogadro constant, N_A. The SI coherent unit of $M(X)$ is kg mol^{-1}. For any elementary entity X, $M(X)$ is related to $m(X)$ through N_A:

$$M(X) = m(X)\, N_A = A_r(X)\, m_u\, N_A. \tag{A1.3}$$

The molar mass constant M_u is defined as

$$M_u = M(^{12}C)/12 \;. \tag{A1.4}$$

These four equations relate the various quantities which are the building blocks of atomic and molar masses and, by extension, are often applied to subatomic and molecular masses.

A1.4.2. Changes of uncertainties

To discuss the implications of Resolution 1 [5.1], we begin with two additional equations taken from the Rydberg relation of atomic physics,

$$hR_\infty = \frac{1}{2} m_e \alpha^2 c \,, \tag{A1.5}$$

where R_∞ is the Rydberg constant, $m_e \equiv m(e)$ is the electron rest mass, α is the fine-structure constant and c is the speed of light in vacuum.

First, it follows from (A1.2) and (A1.5) that for any entity X,

$$\frac{h}{m(X)} = \frac{1}{2} \frac{A_r(e)}{A_r(X)} \frac{\alpha^2 c}{R_\infty} \;. \tag{A1.6}$$

Second, from (A1.3), (A1.4) and (A1.6),

$$\frac{N_{\mathrm{A}}h}{M_{\mathrm{u}}} = \frac{1}{2}A_{\mathrm{r}}(\mathrm{e})\frac{\alpha^2 c}{R_\infty}. \tag{A1.7}$$

The right-hand side of (A1.7), which is traceable to the SI units of time and length, has a relative standard uncertainty of 4.5×10^{-10} [5.3] at the time of the revision of the SI. This relation is key to understanding how the uncertainties of M_{u} and m_{u} were affected by Resolution 1 of the 26th CGPM (2018).

Of the constants appearing in the seven relations shown above, M_{u} (and by extension $M(^{12}\mathrm{C})$), had a fixed numerical value before the SI was revised by the 26th meeting of the CGPM, but no longer. The constants N_{A} and h did not have fixed numerical values prior to the 26th CGPM. (The value of the speed of light in vacuum has been fixed since 1983).

Thus Resolution 1 of the 26th CGPM has had the following consequences to the quantities and measurements discussed above:

1. Relative atomic masses (and their uncertainties) are unaffected. They are dimensionless ratios and thus independent of unit systems. In the field of chemistry, relative atomic masses are often referred to as atomic weights.

2. Determinations of the fine-structure constant have been unaffected.

3a. Neither the value nor the uncertainty of $N_{\mathrm{A}}h/M_{\mathrm{u}}$ were affected by Resolution 1. The value of this combination of constants is still determined from the recommended values for the parameters on the right-hand side of (A1.7), and these are either traceable to SI units of time and length or are pure numbers.

In some scientific papers published prior to the adoption of Resolution 1, the quantity $N_{\mathrm{A}}h/M_{\mathrm{u}}$ has been written as $N_{\mathrm{A}}h(10^3)$, where the factor 10^3 was used as a kind of short-hand to indicate the exact numerical value of M_{u}^{-1} whose SI coherent unit is mol kg^{-1}. This short-hand arose because the mole was defined through the definition of the kilogram combined with an exact numerical value of M_{u} equal to 10^{-3} kg mol^{-1}; but the mole is now defined through a fixed numerical value of N_{A}, whose SI coherent unit is mol^{-1}. Nevertheless, M_{u} may still be taken to be 0.001 kg mol^{-1} as long as the relative standard uncertainty of M_{u}, which is currently 4.5×10^{-10} [5.3], can be neglected in the uncertainty budget of a measurement under discussion.

3b. For no other reason than to bring clarity to the discussion in this subsection, the changes to the value of M_{u} and its uncertainty may be parameterized in terms of a small, dimensionless quantity κ. The molar mass constant M_{u}, instead of being defined as exactly 0.001 kg mol^{-1}, as it was prior to the adoption of Resolution 1, can be accurately derived from the last term of the following relation

$$M_{\mathrm{u}} = \left(0.001\,\mathrm{kg\ mol}^{-1}\right)(1+\kappa) = \frac{R_\infty}{A_{\mathrm{r}}(\mathrm{e})\alpha^2}\left(\frac{2N_{\mathrm{A}}h}{c}\right), \tag{A1.8}$$

where, in the last term, the constants in the final parentheses have exactly defined values.

Due to the principle of continuity when changes are made to the SI, the value of κ is consistent with zero to a standard uncertainty of $u(\kappa) = u_{\mathrm{r}}(R_\infty/(A_{\mathrm{r}}(\mathrm{e})\alpha^2))$, which at present is 4.5 parts in 10^{10}. This uncertainty would be further reduced by improved measurements of the constants involved, α in particular. The accepted values and relative uncertainties of $A_{\mathrm{r}}(\mathrm{e})$, R_∞ and α are the CODATA 2017 recommended values [5.3].

The molar mass constant and the unified atomic mass constant are related by $M_u = m_u N_A$. It follows that, since $u_r(N_A) = 0$, the relative uncertainties of m_u and M_u are identical:

$$u_r(m_u) = u_r(M_u) = u(\kappa). \tag{A1.9}$$

For the case of m_u, whose value has been (and remains) determined by experiment, the adoption of Resolution 1 nevertheless resulted in a reduction of $u_r(m_u)$ by more than a factor of 20 simply by defining h to have a fixed numerical value, although this improved uncertainty does not seem to have any immediate practical benefits.

Finally, in atomic physics it is sometimes necessary to convert between the non-SI units electronvolt (symbol: eV) and the unified atomic mass unit (symbol: u). The correspondence is at present

$$1 \text{ u} \leftrightarrow 931.494\ 102\ 74\ (42) \times 10^6 \text{ eV}\ , \tag{A1.10}$$

where the numerical value of the energy expressed in electronvolts equals the numerical value of $m_u c^2/e$ expressed in joules per coulomb. The quantities c and e have fixed numerical values.

A2. Maintenance of practical realizations

In the past, an experiment capable of determining the value of the Planck constant provided a result of enduring value, even if the experiment was never repeated. Now that similar experiments are used to realize the mass unit, we discuss briefly whether an abbreviated experiment could be used to ensure that the realization remains valid. If we consider the realizations described in Section 2, the basic question is: must routine realizations of primary mass standards be identical to the first such realization? Some considerations are given here.

For realization through a Kibble balance: Assurances are needed that the mechanical and magnetic alignments of the balance remain adequate; that SI traceability is maintained to auxiliary measurements of velocity, gravitational acceleration, current and voltage. Improved technology in these areas opens the possibility of reducing the uncertainty of the realization.

For a realization through the XRCD method, ^{28}Si-enriched, single-crystal silicon ingots were prepared. X-ray interferometers, samples for molar mass measurements, two 1 kg spheres for the density measurement, and many other samples were prepared from each ingot. The spheres are primary mass standards from which the mass unit can be disseminated, but the spheres must be maintained in good condition for periodic monitoring by appropriate methods of the following parameters:

- Surface layers on the silicon spheres by, for example, spectral ellipsometry, X-ray refractometry (XRR), X-ray photoelectron spectrometry (XPS), X-ray fluorescence (XRF) analysis, and infrared absorption;
- Volume of the silicon spheres by, for example, optical interferometry.

These measurements are not onerous and it is estimated that they could be carried out within a few weeks.

In addition, although no known mechanism would change the molar mass of the crystals, re-measurement of the molar mass by improved methods could reduce the uncertainty with which the kilogram definition can be realized by the XRCD method.

Similarly, there is no known mechanism for the edge dimension a(Si) of the unit cell to change with respect to time, but re-measurement of this quantity by combined X-ray and optical interferometry could reduce the uncertainty with which the kilogram definition can be realized by the XRCD method.

Confirmation can be provided by mechanisms of the CIPM MRA, which provide measures of the equivalence of the various realizations.

A3. Maintenance of mass correlation among artefacts calibrated by NMIs or DIs realizing the kilogram (informational)

In the context of the CIPM MRA, an NMI, DI or the BIPM, realizing the mass unit would be able to calibrate mass standards traceable to their own realization only, provided that the laboratory has participated with success in a key comparison as described in section 3.1. However, as long as the uncertainty of a primary realization is significantly larger than the uncertainty of a mass comparison, the uncertainty of a calibration traceable to a single realization would be larger than the uncertainty of a calibration traceable to multiple realizations at least in the case of independent and consistent results.

Laboratories realizing the mass unit might take advantage of the information obtained in key comparisons in order to reduce the mass calibration uncertainty and increase the correlation of mass measurement worldwide. The following simplified example illustrates how the analysis of the key comparison might be modified in order to achieve this.

Assume that a number n of laboratories is realizing the mass unit. These laboratories are labeled $\mathrm{NMI}_1,\ldots,\mathrm{NMI}_n$. As a result of the realization, NMI_i assigns a prior value m_i and an associated standard uncertainty $u(m_i)$ to a stable mass standard S_i with nominal mass 1 kg. In a subsequent key comparison, NMI_i measures the mass difference between the standard S_i and a circulated, stable mass standard S_R. NMI_i reports the measured mass difference Δm_i, the prior mass value m_i and the associated standard uncertainties $u(\Delta m_i)$ and $u(m_i)$.

The key comparison reference value $\hat{m}_\mathrm{R}$ (the mass of the circulated standard S_R) and highly correlated posterior values $\hat{m}_i$ of the mass standards S_i are obtained as the weighted least squares solution to the model

$$\begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \\ \Delta m_1 \\ \Delta m_2 \\ \vdots \\ \Delta m_n \end{pmatrix} \doteq \begin{pmatrix} 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \\ 1 & 0 & \ldots & 0 & -1 \\ 0 & 1 & \ldots & 0 & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 & -1 \end{pmatrix} \begin{pmatrix} \hat{m}_1 \\ \hat{m}_2 \\ \vdots \\ \hat{m}_n \\ \hat{m}_R \end{pmatrix} \qquad \text{(A3.1)}$$

(The symbol $\doteq$, also used in [5.3], indicates that an input datum of the type on the left-hand side is ideally given by the expression on the right-hand side containing adjusted quantities.)

In the subsequent dissemination of mass unit, NMI_i uses the stable mass standard S_i as reference, but with the posterior value $\hat{m}_i$ and associated standard uncertainty $u(\hat{m}_i)$ rather than the prior value m_i and associated standard uncertainty $u(m_i)$.

For simplicity, the above example is based on the assumption that stable mass standards are available. Such standards were not available in the past, and they may not be available in the future either. However, as long as the changes in mass standards are predictable with an uncertainty smaller than the uncertainty of the realization of the mass unit, a procedure similar to the one described, but which takes into account the instability of the mass standards, will provide posterior mass values with smaller uncertainties and higher correlations than those of the prior values.